Hybrid DFSF-BP Equalization for ATSC DTV Receivers

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***Abstract*—Severe intersymbol interference (ISI) is one of the main obstacles for reliable signal reception in ATSC DTV systems. Decision feedback equalizers (DFEs) are commonly used to suppress the ISI. However, DFEs may suffer from error propagation due to incorrect symbol decisions from the symbol slicer. This phenomenon deteriorates the performance even more when the post-cursor ISI is strong. In order to reduce error propagation, we present a novel hybrid equalization scheme for ATSC channels. The proposed scheme consists of an adaptive decision feedback sparsening filter (DFSF), and an iterative maximum *a posteriori* (MAP) equalizer based on the belief propagation (BP) algorithm. In the first stage, instead of removing all the ISI from post cursors, the DFSF employs a modified feedback filter which leaves the strongest post-cursor ISI taps uncorrected. As a result, a long ISI channel is equalized to a sparse channel having only a small number of nonzero taps. In the second stage, a belief propagation algorithm is applied to mitigate the residual ISI. Since the channel is typically time-varying and suffers from Doppler fading, the DFSF is adapted using the least mean square (LMS) algorithm, such that the amplitude and the locations of the nonzero taps of the equalized sparse channel appear to be fixed. As such, the channel appears to be static during the second stage of equalization which consists of the BP detector. Simulation results demonstrate that the proposed scheme outperforms the traditional DFE in symbol error rate, under both static channels and dynamic ATSC channels.**

***Index Terms*—ATSC DTV, belief propagation algorithm, channel sparsening, intersymbol interference.**

I. Introduction

In Advanced Television Systems Committee (ATSC) digital television (DTV) systems [1], terrestrial channels often suffer from strong multipath distortion. The duration of the channel impulse response (CIR) can span hundreds of symbol periods. The long and large pre-cursors and post-cursors of the channel impose great challenges to performing reliable equalization of the 8-vestigial sideband (VSB) signals. To combat the severe intersymbol interference (ISI) by the multipath channel, decision feedback equalizers (DFEs) [2] are commonly used in DTV receivers. It is well known that DFEs undergo error propagation which results in bursty errors. When the symbol decisions from the slicer output are incorrect, the feedback filter fails to subtract off accurate residual ISI from the feedforward filter output. The incorrect decisions exacerbate the ISI when large post-cursors are present in the combined response of the channel and feedforward equalizer since the largest taps in the feedback path will contribute the most unintended ISI.

In order to reduce error propagation, a decision feedback sparsening filter (DFSF) combined with a belief propagation (BP) equalizer [3] was proposed in [4]. First, the DFSF conditions the channel to a sparse channel with only a few nonzero taps, the number of which is specified by the system designer. By setting the largest taps in the feedback path to zero, residual ISI is intentionally present at the output of the DFSF. By specifically zeroing the largest taps in the feedback filter (corresponding to the taps that contribute the most ISI), there is less chance of introducing unintended ISI in the event of symbol decision errors. In the next stage of the receiver, the residual sparse ISI is compensated by the BP equalizer, which provides near-optimal error performance, with the complexity depending on the number of system designer-specified nonzero taps in the effective channel. This BP equalizer has been shown to be effective and feasible for a sparse channel with only a few nonzero taps [5], [6].

In this paper, we present a hybrid DFSF-BP equalization scheme for DTV receivers. Our idea is conceived from the scheme in [4], though the major difference from previous work is that our scheme targets 8-VSB-based ATSC DTV systems and is suitable for channels that experience Doppler fading; [4] only considers static channels and BPSK modulation. In our proposed scheme, the DFSF is designed to adapt to the time-varying channel using the least mean square (LMS) algorithm, such that the channel can be assumed to be fixed after the DFSF processing. Thus the second-stage BP equalizer can benefit from the fixed channel by reducing the implementation complexity. We assess the error performance of the proposed scheme by simulation under static/dynamic ATSC DTV channel models. Simulation results show that the proposed scheme outperforms the traditional DFE in both static and dynamic environments.

II. System Model

The system model is shown in Fig. 1. We assume that the data symbols $x[k]$ are Reed-Solomon (RS)-encoded, interleaved, and trellis-encoded symbols, drawn i.i.d. from the 8 level pulse amplitude modulation constellation $(\pm 1, \pm 3, \pm 5, \pm 7)$ [1]. The data symbols $x[k]$, with a variance of σ_x^2, are transmitted through an ISI channel at a symbol rate of 10.76 MHz.

We assume that a squared root raised cosine (SRRC) filter is applied as the transmitter filter. The received signal is processed by a matched filter, and then sampled at the symbol rate. The equivalent discrete-time CIR at time k

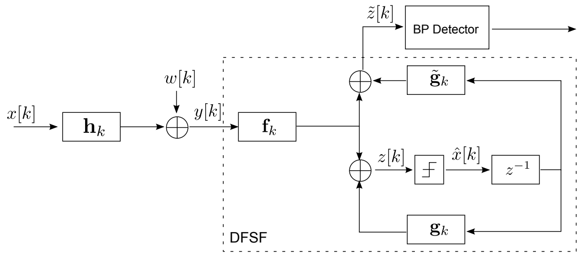

Figure 1. System model

which includes the effects of pulse shaping is described as $\mathbf{h}_k = [h_k[0], h_k[1], ..., h_k[L_h - 1]]^T$, where L_h is the channel length. The received signal at time k can be expressed as

$$y[k] = \sum_{i=0}^{L_h-1} h_k[i]x[k-i] + w[k],$$

where $w[k]$ is additive white Gaussian noise with variance σ_n^2.

The DFSF consists of two parts: a regular DFE and a modified feedback filter $\tilde{\mathbf{g}}_k$. In the DFE, the feedforward filter and the feedback filter are denoted as $\mathbf{f}_k$ and $\mathbf{g}_k$, respectively. Note that since the channel $\mathbf{h}_k$ is time-varying, the filters are also time dependent. The filters $\mathbf{f}_k$, $\mathbf{g}_k$, and $\tilde{\mathbf{g}}_k$ have discrete finite impulse responses, which are expressed as $\mathbf{f}_k = [f_k[0], f_k[1], ..., f_k[L_f - 1]]^T$, $\mathbf{g}_k = [g_k[0], g_k[1], ..., g_k[L_g - 1]]^T$, and $\tilde{\mathbf{g}}_k = [\tilde{g}_k[0], \tilde{g}_k[1], ..., \tilde{g}_k[L_{\tilde{g}} - 1]]^T$. The input signal for the slicer is given by

$$z[k] = \sum_{i=0}^{L_f-1} f_k[i]y[k-i] + \sum_{i=0}^{L_g-1} g_k[i]\hat{x}[k-i-1],$$

where $\hat{x}[k]$ is the tentative decisions from the slicer. The modified feedback filter only suppresses partial ISI using the tentative decisions. The output signal of the DFSF is

$$\tilde{z}[k] = \sum_{i=0}^{L_f-1} f_k[i]y[k-i] + \sum_{i=0}^{L_g-1} \tilde{g}_k[i]\hat{x}[k-i-1]. \quad (1)$$

Assuming that the tentative decisions are correct, $\hat{x}[k]$ is the delayed version of symbol input $x[k]$. Then (1) can be simplified as

$$\tilde{z}[k] = \sum_{i=0}^{L_c-1} c_k[i]x[k-i] + n[k],$$

where $c_k[i]$ contains the coefficients of the combined response of the CIR and DFSF at time k, and $n[k]$ is the DFSF output noise. We denote the combined response of the CIR and DFSF as $\mathbf{c}_k$ with the channel length L_c. The system designer chooses the desired number of nonzero taps D in $\mathbf{c}_k$, and the modified feedback filter $\tilde{\mathbf{g}}_k$ is obtained from $\mathbf{g}_k$ by setting its $(D-1)$ largest taps to zero.

In the second stage, the BP equalizer will compensate for the D nonzero taps. Since the complexity of the BP equalizer depends the number of nonzero taps of the CIR, we can trade performance for complexity of the overall equalization scheme by choosing different values of D. With small values of D, most ISI is canceled by the DFSF, and thus the burden on the complexity of the BP equalizer is reduced. However, it is more likely that the unintended ISI is introduced in the feedback loop, which will cause error propagation. A extreme case is to choose $D = 1$ in which case the DFSF is equivalent to the regular DFE, and the BP equalizer is reduced to a slicer. To minimize the incorrect feedback of ISI components, the system designer can increase the value of D so that more ISI components are processed by the near-optimal BP equalizer. On the other hand, since the complexity of the BP equalizer increases exponentially with the nonzero taps of the CIR, D should be constrained to a small enough value that permits practical implementation.

III. Proposed Equalization Scheme

In DTV systems, the channel typically experiences Doppler fading [2]. In [4], the channel is assumed to be fixed and known perfectly to the receiver, and without modification the DFSF-BP scheme of [4] cannot be directly applied here. We now extend the DFSF-BP equalization scheme to an adaptive implementation suitable for use in time-varying channels. Recall that the receiver under consideration is a two-stage receiver, where in the first stage an equalizer partially equalizes the channel, and the residual ISI is compensated by a belief propagation detector in the second stage.

The equalizer operates in two modes: (A) a startup mode, where the DFSF taps are initialized based on training data or other channel sounding techniques, and (B) a tracking mode where the DFSF adapts to the time-varying channel using a decision-directed approach. We make the assumption that the channel is approximately static during the startup mode, and as such we can directly apply the results of [4] to initialize the DFSF taps.

A. Startup Mode

We adopt the minimum mean-squared error (MMSE) criterion to design the feedforward filter $\mathbf{f}_k$, the feedback filter $\mathbf{g}_k$, the modified feedback filter $\tilde{\mathbf{g}}_k$. Here we assume that the channel is approximately static over a block of length L_f. The filters $\mathbf{f}_k$ and $\mathbf{g}_k$ are designed to have coefficients given by the classical MMSE-DFE which has solution

$$\mathbf{f}_k = \sigma_x^2 \left(\sigma_x^2 \mathbf{H}_k (\mathbf{I}_{L_c} - \mathbf{\Sigma}^T\mathbf{\Sigma})\mathbf{H}_k^T + \sigma_n^2 \mathbf{I}_{L_f}\right)^{-1} \mathbf{H}_k \mathbf{e}, \quad (2)$$

$$\mathbf{g}_k = -\mathbf{\Sigma}\mathbf{H}_k^T\mathbf{f}_k, \quad (3)$$

where $\mathbf{H}_k$ is the channel convolution matrix denoted as

$$\mathbf{H}_k = \begin{bmatrix} h_k[0] & h_k[1] & \dots & h_k[L_h-1] & 0 & 0 & \dots \\ 0 & h_k[0] & h_k[1] & \dots & h_k[L_h-1] & 0 & \dots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \\ 0 & & & & & & \\ 0 & \dots & 0 & h_k[0] & h_k[1] & \dots & h_k[L_h-1] \end{bmatrix},$$

and

$$\mathbf{\Sigma} = [\mathbf{0}_{L_g\times(\Delta+1)} \quad \mathbf{I}_{L_g} \quad \mathbf{0}_{L_g\times(L_h+L_f-L_g-\Delta-2)}],$$

and

$$\mathbf{e} = \left[\underbrace{0, \ldots, 0}_{\Delta},\ 1,\ \underbrace{0, \ldots, 0}_{L_f + L_h - \Delta - 2} \right]^T,$$

where Δ is the decision delay of the DFE slicer. Next, $\tilde{\mathbf{g}}_k$ is chosen to be equal to $\mathbf{g}_k$ but with the $(D-1)$ largest taps set to zero to mitigate the effects of errors out of the symbol slicer. Consequently, the coefficients of the combined response of the CIR and DFSF $\mathbf{c}_k$ can be calculated as

$$c_k[i] = h_k[i] \star f_k[i] + \tilde{g}_k[i - \Delta - 1], \tag{4}$$

where $(\star)$ denotes the convolution operation. Under the assumption of correct decisions at the output of the slicer, $\mathbf{c}_k$ corresponds to an impulse response with D nonzero coefficients.

To compute the initial MMSE equalizer settings at the start of transmission, we make the assumption that the receiver either has knowledge of the initial channel coefficients and can compute the initial equalizer setting via (2) and (3); alternatively, the receiver may use training data and a trained algorithm like LMS to directly adapt the equalizer coefficients $\mathbf{f}_k$, $\mathbf{g}_k$, and $\tilde{\mathbf{g}}_k$ to the initial MMSE solution.

Finally, we make the assumption that the startup mode is completed at time $k = 0$, so $\mathbf{f}_0$ and $\mathbf{g}_0$ are the MMSE-DFE coefficients. Furthermore, $\tilde{\mathbf{g}}_0$ is a zeroed version of $\mathbf{g}_0$, and the initial combined response $\mathbf{c}_0$ can be computed via (4).

B. Tracking Mode

We now develop an approach for using the DFSF with time-varying channels that experience Doppler. One approach is to continuously adapt (using, for example, decision-directed LMS) the coefficients $\mathbf{f}_k$, $\mathbf{g}_k$, and $\tilde{\mathbf{g}}_k$ to track the MMSE solution at time k for any channel $\mathbf{h}_k$. Then, however, the effective channel $\mathbf{c}_k$ observed by the BP detector would also be time-varying since the intentional residual ISI terms (i.e. those taps which get zeroed in $\tilde{\mathbf{g}}_k$) change in both amplitude and location as the channel changes. Such a time-varying effective channel would lead to a BP detector which is significantly more complicated than a conventional static BP detector.

Consequently, instead of adapting $\mathbf{f}_k$, $\mathbf{g}_k$, and $\tilde{\mathbf{g}}_k$ to track the MMSE solution, we adapt them so the combined effective response $\mathbf{c}_k$ appears static to the BP detector. That is, we design an algorithm to adapt $\mathbf{f}_k$, $\mathbf{g}_k$, and $\tilde{\mathbf{g}}_k$ so that the combined response is equal to the initial combined response $\mathbf{c}_0$ in the MMSE sense. As such, the equalizer coefficients will coincide with the MMSE-DFE setting at startup, and will then gradually drift away from that setting as the channel changes and the equalizer adjusts to maintain a static effective channel.

As the channel changes significantly, the feedback filter $\tilde{\mathbf{g}}_k$ may have taps that grow large in attempt to keep the combined response equal to $\mathbf{c}_0$. Recall that our motivation for using the hybrid DFSF-BP scheme is to mitigate error propagation by keeping taps in the feedback path small, and relying on the more sophisticated BP algorithm to compensate for the significant ISI terms. Consequently, it may prove beneficial to periodically reset the DFSF if the channel drifts to a situation resulting in large taps in $\tilde{\mathbf{g}}_k$, which can be accomplished by repeating the startup procedure.

Algorithm 1 DD-LMS algorithm for DFSF

Parameters:
Feedforward filter coefficients at time k: $\mathbf{f}_k$
Feedback filter coefficients at time k: $\mathbf{g}_k$
Modified feedback filter coefficients at time k: $\tilde{\mathbf{g}}_k$
Combined response of the CIR and DFSF at time k: $\mathbf{c}_k$
Received data vectors at time k:
$\mathbf{y}_k = [y[k], y[k-1], \ldots, y[k - L_f + 1]]^T$
Tentative decisions at time k:
$\hat{\mathbf{x}}_{k,L_g} = [\hat{x}[k], \hat{x}[k-1], \ldots, \hat{x}[k - L_g + 1]]^T$
$\hat{\mathbf{x}}'_{k,L_c} = [\hat{x}[k], \hat{x}[k-1], \ldots, \hat{x}[k - L_c + 1]]^T$
Step size for updating $\mathbf{f}_k$: μ_f
Step size for updating $\mathbf{g}_k$: μ_g
Step size for updating $\tilde{\mathbf{g}}_k$: $\mu_{\tilde{g}}$
Decision-directed error at time k: $e[k]$
Error between the actual DFSF output and the expected DFSF output at time k: $\tilde{e}[k]$

Startup mode:
Set the initial coefficients $\mathbf{f}_0$ and $\mathbf{g}_0$ using (2) and (3).
Set the initial coefficients $\tilde{\mathbf{g}}_0$ to $\mathbf{g}_0$ but with the $(D-1)$ largest taps set to zero.
Set the initial coefficients $\mathbf{c}_0$ using (4).

Tracking mode:
Update the coefficients at each time instant:

$$\begin{aligned} \mathbf{f}_{k+1} &= \mathbf{f}_k - \mu_f e[k] \mathbf{y}_k \\ \mathbf{g}_{k+1} &= \mathbf{g}_k - \mu_g e[k] \hat{\mathbf{x}}_{k-1,L_g} \\ \tilde{\mathbf{g}}_{k+1} &= \tilde{\mathbf{g}}_k - \mu_{\tilde{g}} \tilde{e}[k] \hat{\mathbf{x}}_{k-1,L_g}, \end{aligned}$$

where

$$\begin{aligned} e[k] &= \mathbf{f}_k^T \mathbf{y}_k + \mathbf{g}_k^T \hat{\mathbf{x}}_{k-1,L_g} - \hat{x}[k] \\ \tilde{e}[k] &= \mathbf{f}_k^T \mathbf{y}_k + \tilde{\mathbf{g}}_k^T \hat{\mathbf{x}}_{k-1,L_g} - \mathbf{c}_0^T \hat{\mathbf{x}}_{k,L_c}. \end{aligned}$$

Once the initial $\mathbf{f}_k$, $\mathbf{g}_k$, and $\tilde{\mathbf{g}}_k$ are obtained, we adapt the DFSF to the channel using the LMS algorithm in decision-directed mode. The decision-directed LMS (DD-LMS) algorithm for DFSF is listed in Algorithm 1. As mentioned above, the adaptive DFSF tracks the time-varying channel and keeps the combined response of the CIR and DFSF $\mathbf{c}_k$ fixed from the perspective of the BP detector, which explains the presence of $\mathbf{c}_0$ in the error term $\tilde{e}[k]$ used in updating $\tilde{\mathbf{g}}_k$. By adapting the DFSF to maintain a static combined response, the implementation of the BP detector is drastically simplified.

In the second stage, we adopt an iterative equalizer based on the BP algorithm, which has been widely used for iterative decoding of low-density parity-check (LDPC) codes [7]. Although the complexity of the BP equalizer increases only with the number of nonzero channel coefficients, the direct implementation of the BP equalizer is still impractical due

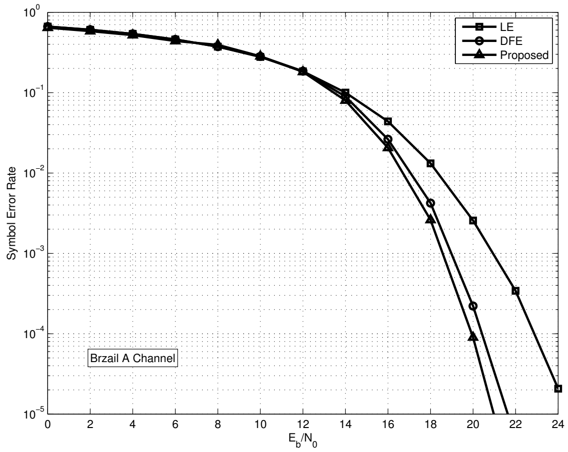

Figure 2. SER performance in Brazil A channel

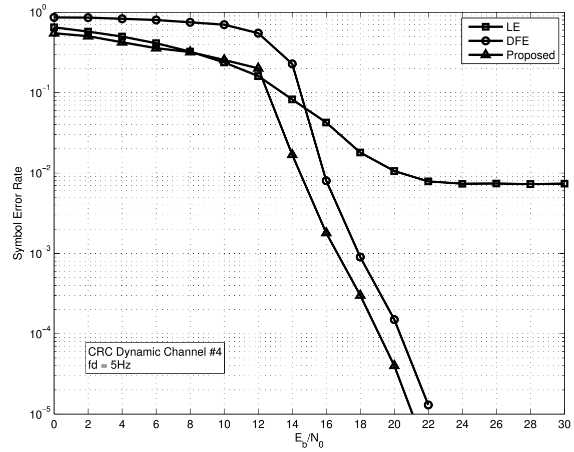

Figure 3. SER performance in CRC #4 channel with 5Hz Doppler shift

to the prohibitively high complexity even for sparse channels where the number of nonzero coefficients is on the order of 10 [5]. To make use of the near-optimal BP equalizer, the number of nonzero taps D must be limited to a small number. This permits the design effort of the BP detector to be greatly mitigated while maintaining near-optimal performance.

IV. Numerical Results and Remarks

The performance of the proposed equalization scheme is evaluated by simulations in terms of symbol error rate (SER) versus signal-to-noise ratio per bit E_b/N_0. The static channel profile used for the simulation is Brazil A channel which is the severe indoor channel used for the Laboratory Test in Brazil, and the dynamic channel profile is CRC channel #4. The channel profile details are listed in [2]. Note that the attenuation of Path 5 of the CRC Dynamic #4 channel is denoted as *threshold of visibility* (TOV). We set the attenuation of Path 5 to 3 dB for different Doppler shifts [8]. According to the ATSC DTV standard, the symbol rate is 10.76 MHz and the roll-off factor of the pulse shaping filter is 11.5%. The performance of a classical MMSE linear equalizer (LE) and classical DFE is also simulated to compare with the proposed scheme. The LE employs an FIR with 800 taps. The DFE consists of 400 feedforward taps and 400 feedback taps. The filter lengths for the DFSF are $L_f = L_g = L_{\tilde{g}} = 400$, and the updating steps are $\mu_f = \mu_g = \mu_{\tilde{g}} = 10^{-5}$. The combined response of the CIR and DFSF $\mathbf{c}_k$ contains $D = 3$ nonzero taps. The number of iterations in BP detection is 5.

We assess the proposed scheme in static ATSC channels. Fig. 2 shows the SER performance for Brazil A channel. It is shown that at a SER of 10^{-5}, the proposed scheme exhibits performance about 1 dB better than the traditional DFE.

The SER performance for dynamic channel is shown in Fig. 3, for Doppler shift $f_d = 5$ Hz. We can see that the proposed scheme can successfully track the channel changes. It is also demonstrated that the proposed scheme provides around 1 dB performance gain over the DFE at a SER of 10^{-5}.

V. Conclusions

We present a hybrid equalization scheme for ATSC DTV systems. In our scheme we first use a decision feedback sparsening filter (DFSF) to equalize the time-varying channel to a static sparse channel with only a few nonzero taps. Then a near-optimal belief propagation (BP) equalizer is adopted to further compensate the residual ISI. Since the DFSF uses a modified feedback filter which only cancels the ISI from less significant taps, and leaves the ISI from the dominant taps for the BP equalizer, there is less chance to introduce unintended ISI in the feedback loop, and thus the impact of error propagation to the overall system is reduced. To address the Doppler shift effect in the practical ATSC channels, the DFSF is designed adaptively using the least mean square (LMS) algorithm to track the time-varying channel. The simulation results under both static and dynamic channels demonstrate that the proposed scheme outperforms the traditional DFE in terms of symbol error rate.

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