Design of a Maximum-Likelihood Detector for Cooperative Communications in Intersymbol Interference Channels

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ABSTRACT

Recently, cooperative communication has attracted a lot of attention for its potential to increase spatial diversity. However, limited attention has been paid to the physical layer and implementation issues. In this paper, we investigate the feasibility of building optimal detectors for cooperative communications in intersymbol interference channels. A novel system model with the amplify-and-forward half-duplex relay is first introduced. Subsequently, we propose an optimal detector for receiver design which can be realized with a whitening filter and maximum likelihood sequence estimator (MLSE) based on the Viterbi algorithm. The implementation of the proposed detector is discussed, and its complexity is analyzed based on an FPGA implementation result. Numerical simulations demonstrating the performance of the detector are provided.

Categories and Subject Descriptors

B.7.1 [**Integrated Circuits**]: Types and Design Styles—*Algorithms implemented in hardware*; C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design—*Wireless communication*

General Terms

Design

1. INTRODUCTION

Recently, cooperative diversity [9, 6] and relay networks [5] have attracted a lot of attention for their ability to exploit increased spatial diversity available at distributed antennas on other nodes in the system. By intelligent cooperation among nodes in the network which may only have a single antenna, a virtual multiple antenna system can be formed. However, most of the existing work has largely come from the information theory and coding communities, and with a few exceptions (e.g. [7]) very little research has yet been conducted into the physical layer and implementation issues of relaying and cooperation.

In this paper, we investigate the feasibility of building optimal detectors for half-duplex relays in frequency selective channels encountered in practice. While a variety of forwarding protocols have been previously proposed, we will consider amplify-and-forward (AF) for its simplicity and reduced implementation costs. Frequency selective channels are an inevitable impairment in wideband communication systems [8], and such channels cause the receiver to observe the superposition of multiple delayed reflections of the transmitted signal, resulting in intersymbol interference (ISI). We will consider the optimistic case where the receiver has perfect channel knowledge, and we will address the design and complexity of an optimal detector for such a system which can be realized with a whitening filter and maximum likelihood sequence estimator (MLSE). After developing a system model for the case of AF relays in ISI channels, we will present an optimal detector realization based on the Viterbi algorithm. We will then present an FPGA-based implementation of the detector and analyze the complexity. Finally, we conclude with numerical simulations demonstrating performance of the detector.

2. SYSTEM MODEL

A three-node cooperative communications system model is shown in Figure 1. Both source and relay can be considered as mobile users, and each has only one antenna. However, the relay can receive the "overheard" information when the source transmits data to the destination, and then forward it to the destination for diversity combining. Since the channels from source and relay to destination are statistically independent, the three-node cooperative communication scheme effectively forms spatial diversity. We assume that a source transmits a continuous stream of data to a destination, and an AF relay assists the source by amplifying and forwarding the data to the destination. A half-duplex relay is assumed which receives for p symbol periods, and then transmits for p symbol periods. The relay repeats these two tasks alternatively.

The source sends signal $\boldsymbol{x} = [x[0], x[1], \dots x[N-1]]^T \in \mathbb{C}^N$, where N is the number of transmitted symbols. Each channel is modeled as a linear time-invariant finite impulse response (FIR) filter together with complex additive white Gaussian noise (AWGN). We assume that the effect of pulse shaping is included in channels. The source-destination, source-relay and relay-destination channel impulse responses are denoted by $\boldsymbol{h}_{sd}$, $\boldsymbol{h}_{sr}$, $\boldsymbol{h}_{rd}$, respectively, and they have

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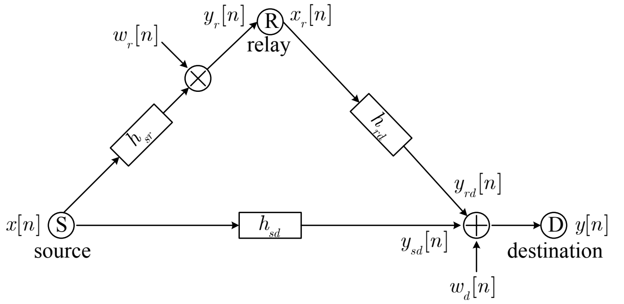

Figure 1: System model

corresponding channel lengths L_{sd}, L_{sr} and L_{rd} (e.g. $\boldsymbol{h}_{sd} = [h_{sd}[0], h_{sd}[1], \ldots h_{sd}[L_{sd}-1]]^T$). $\boldsymbol{w}_r$ and $\boldsymbol{w}_d$ are AWGN with variance σ_r^2 and σ_d^2, respectively.

The destination combines two signals from the source and the relay. For the source-relay-destination link, we have $\boldsymbol{y}_r = \mathbf{H}_{sr}\boldsymbol{x} + \boldsymbol{w}_r$, $\boldsymbol{x}_r = \boldsymbol{\Gamma}\boldsymbol{y}_r$, where $\mathbf{H}_{sr} \in \mathbb{C}^{(N+L_{sr}-1)\times N}$ is the complex Toeplitz channel convolution matrix defined by $[\mathbf{H}_{sr}]_{i,j} = h_{sr}[i-j]$. $\boldsymbol{y}_r \in \mathbb{C}^{N+L_{sr}-1}$ is the signal received by the relay, $\boldsymbol{x}_r \in \mathbb{C}^{N+L_{sr}-1}$ is the signal transmitted from the relay and $\boldsymbol{\Gamma} \in \mathbb{C}^{(N+L_{sr}-1)\times(N+L_{sr}-1)}$ denotes the relay matrix. The function of $\boldsymbol{\Gamma}$ is to impose the half-duplex constraint by copying and removing p symbols from $\boldsymbol{y}_r$ (receiving), scaling these removed symbols by a factor β (amplifying), and then inserting the scaled symbols to $\boldsymbol{y}_r$ during the next p symbol periods (forwarding), recursively. Thus $\boldsymbol{\Gamma}$ is given by

$$\boldsymbol{\Gamma} = \beta \mathbf{I}_{\left(\frac{N+L_{sr-1}}{2p}\right)} \otimes \begin{bmatrix} \mathbf{0}_{p\times p} & \mathbf{0}_{p\times p} \\ \mathbf{I}_{p\times p} & \mathbf{0}_{p\times p} \end{bmatrix} \quad (1)$$

where $\otimes$ denotes Kronecker product. Here we implicitly require that $N + L_{sr} - 1$ is divisible by $2p$.

The received signal at the destination is expressed as

$$\begin{aligned} \boldsymbol{y} &= \overbrace{\mathbf{H}_{sd}\boldsymbol{x}}^{\text{from source}} + \overbrace{\mathbf{H}_{rd}\boldsymbol{x}_r}^{\text{from relay}} + \boldsymbol{w}_d \\ &= \underbrace{(\mathbf{H}_{sd} + \mathbf{H}_{rd}\Gamma\mathbf{H}_{sr})}_{\triangleq \mathbf{H}_{eff}}\boldsymbol{x} + \underbrace{(\mathbf{H}_{rd}\Gamma\boldsymbol{w}_r + \boldsymbol{w}_d)}_{\triangleq \boldsymbol{w}_{eff}} \end{aligned} \quad (2)$$

where $\boldsymbol{w}_{eff} \sim \mathcal{CN}(\mathbf{0}, \sigma_d^2\mathbf{I} + \sigma_r^2\mathbf{H}_{rd}\boldsymbol{\Gamma}\boldsymbol{\Gamma}^H\mathbf{H}_{rd}^H)$. For the matrix dimensions to be compatible, we require that $L_{sd} = L_{sr} + L_{rd} - 1$; if this is not satisfied, we can append zeros to the appropriate matrix without loss of generality. Note that $\boldsymbol{w}_{eff}$ is colored, not white, since the AWGN on the source-relay link get amplified and forwarded over the relay-destination channel which introduces coloring of the noise.

From (1), the relay matrix $\boldsymbol{\Gamma}$ has a repetitive structure with a period of $2p$. Accordingly, the channel matrix $\mathbf{H}_{eff}$ shows the same structure as $\boldsymbol{\Gamma}$. Consequently $\mathbf{H}_{eff}$ is a periodically time-varying channel which consists of $2p$ sets of different channels. In the following section, we will introduce a ML detector to estimate transmitted symbols at the receiver end for the ISI channel model prescribed above.

3. PROPOSED ML DETECTOR

Assuming that the receiver can acquire perfect channel knowledge, MLSE can be employed by searching for the minimum Euclidean distance between observed signal and any given transmitted signals. The Viterbi Algorithm (VA) [10] is an efficient technique for solving the minimum distance problem, and its implementation has been investigated extensively [3, 2, 1]. However, traditional Viterbi detector can only address time-invariant channels. A modified Viterbi detector is proposed to deal with periodically time-varying channels induced by the half-duplex relay. Furthermore, since the minimum Euclidean distance is not optimal under colored Gaussian noise, we employ a whitening filter before detection, which was shown in [4] to be optimal. The block diagram of our design is given in Figure 2. After the whitening filter, the received signal is fed into a modified Viterbi detector which consists of a branch metric unit (BMU) , an add-compare-select unit (ACS) and a survivor-path memory unit (SMU). The details of each component in the ML detector are discussed as follows.

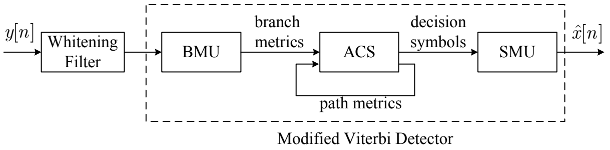

Figure 2: Block diagram of the proposed detector

3.1 Whitening Filter

To whiten the noise, we first factor the noise covariance matrix as:

$$\mathbf{G}\mathbf{G}^H = \sigma_d^2\mathbf{I} + \sigma_r^2\mathbf{H}_{rd}\boldsymbol{\Gamma}\boldsymbol{\Gamma}^H\mathbf{H}_{rd}^H.$$

where $\mathbf{G}$ is the Cholesky factorization of the covariance. Then, by filtering the received signal with $\mathbf{G}^{-1}$ (i.e. by forming $\mathbf{G}^{-1}\boldsymbol{y}$), the noise becomes whitened since the covariance of the filtered noise $\mathbf{G}^{-1}\boldsymbol{w}_{eff}$ is given by

$$E\left[(\mathbf{G}^{-1}\boldsymbol{w}_{eff})(\mathbf{G}^{-1}\boldsymbol{w}_{eff})^H\right] = \mathbf{G}^{-1}E[\boldsymbol{w}_{eff}\boldsymbol{w}_{eff}^H](\mathbf{G}^{-1})^H = \mathbf{I}$$

After applying the whitening filter, the received symbols become $\boldsymbol{y}' = \mathbf{G}^{-1}\mathbf{H}_{eff}\boldsymbol{x} + \mathbf{G}^{-1}\boldsymbol{w}_{eff}$. Ignoring end effects (or, equivalently, taking the block length $N \to \infty$), $\mathbf{G}^{-1}$ follows the same repetitive structure as $\boldsymbol{\Gamma}$, and thus also shows periodically time-varying property. It naturally defines $2p$ sets of coefficients denoted as $\boldsymbol{g}_0$, $\boldsymbol{g}_1 \ldots \boldsymbol{g}_{2p-1}$. Without loss of generality, letting the start time $t = 0$, coefficients adopted by the whitening filter at certain time instant $t = n$ are given by $\boldsymbol{g}_n = \boldsymbol{g}_{\text{mod}(n,2p)}$, where $\text{mod}(n, 2p)$ is the modulus reminder.

3.2 The Modified Viterbi Detector

The effective channel matrix $\mathbf{G}^{-1}\mathbf{H}_{eff}$ maintains the periodically time-varying property due to the similar structures of $\mathbf{G}^{-1}$ and $\mathbf{H}_{eff}$. Correspondingly, a time-varying trellis diagram can be developed to describe state transitions. As a result, the output symbols along the trellis path are not only related to state transitions but also the current time instant. Thus the branch metrics calculation is modified as

$$BM_{i,j,n} = \|y'[n] - s_{i,j,n}\|^2 \quad (3)$$

where $y'[n]$ is the symbol from the whitening filter at the time instant $t = n$, $BM_{i,j,n}$ and $s_{i,j,n}$ are branch metric and output symbol from state i to state j at $t = n$, respectively. Since $\mathbf{G}^{-1}\mathbf{H}_{eff}$ defines $2p$ sets of effective channel

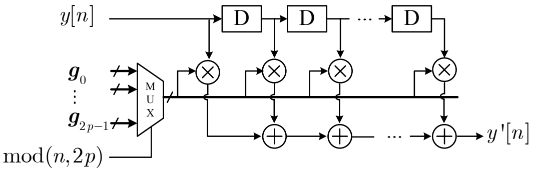

Figure 3: Whitening filter structure

coefficients, each of which is applied to the trellis every $2p$ symbol periods, we have $s_{i,j,n} = s_{i,j,\text{mod}(n,2p)}$. In order to calculate $BM_{i,j,n}$ at $t = n$, we can precompute $2p$ sets of output symbols, $s_{i,j,0}$,$s_{i,j,1}$,. . . ,$s_{i,j,2p-1}$, and replace $s_{i,j,n}$ in (3) with the corresponding set at $t = n$.

ACS recursively computes path metrics and decision bits,

$$PM_{j,n} = \min_{\{i\}} [PM_{i,n-1} + BM_{i,j,n}] \tag{4}$$

where $PM_{j,n}$ denotes the path metric at state j when $t = n$ and i corresponds to the previous state of j. The path metric for each state is updated for the next iteration, and the decision bits $d_{j,n}$ indicating the survivor path for state j at $t = n$ are recorded in the memory of SMU.

Register exchange (RE) and traceback (TB) are two basic approaches to record survivor paths. RE is widely considered a suitable approach for short constraint length, while TB is preferred in the design of large constraint length, high performance Viterbi detector [3]. With respect to the received signal, the length of channel impulse response follows the larger channel length between the source-destination link and source-relay-destination link. Then the length may be extended by the whitening filter. Hence, let L_{eff} be the effective channel length with respect to the signal after the whitening filter, we have

$$L_{eff} \geq \max(L_{sd}, L_{sr} + L_{rd} + p - 1).$$

It suggests that the constraint length for the MLSE can be very large under a practical situation, for example, when the relay cooperation period p is large. Thus TB approach is more appropriate for our design.

4. IMPLEMENTATION AND COMPLEXITY ANALYSIS

A ML detector for QPSK system is considered in this study. We restrict the constraint length to 3 by selecting proper channel and relay parameters, i.e. $L_{sd} = 3$, $L_{sr} = L_{rd} = 2$, $p = 1$. The number of states in the trellis is $4^3 = 64$. Based on fixed-point simulation, the received symbols are quantized to 6 signed bits for real and imaginary parts respectively.

The whitening filter is an infinite impulse response (IIR) filter in practice, but the impulse response is typically truncated. Thus it can be carried out using an FIR filter coupled with a multiplexer as in Figure 3, where multipliers and adders are for complex operations. The control signal $\text{mod}(n, 2p)$ decides the current phase from 0 to $2p - 1$, and the coefficients for $t = n$ are applied to the filter.

BMU calculates Euclidean distance of $y'[n]$ and $s_{i,j,n}$ for each state transition by

$$\|y'[n] - s_{i,j,n}\|^2 = \text{Re}(y'[n] - s_{i,j,n})^2 + \text{Im}(y'[n] - s_{i,j,n})^2$$

which includes 3 additions and 2 multiplications. The multiplexer for BMU share the same control signal with the one for the whitening filter. On the other hand, the whitening filter and BMU must be strictly synchronized since the coefficients they use are decided by the current phase.

Table 1: FPGA resource usage of proposed detector

Target device	Altera Stratix II EP2S180
Combinational ALUTs	11074 out of 143520 (8%)
Dedicated logic registers	1618 out of 143520 (1%)
Total block memory bits	8192 out of 9383040 (<1%)
DSP block 9-bit elements	512 out of 768 (67%)

For QPSK modulation, the 64-state trellis can be decomposed into 16 4-state subtrellises. Each subtrellis is implemented using radix-4 unit [1] consisting of four 4-way ACS units. Each 4-way ACS unit includes 4 adders, a 4-way comparator and a 4-to-1 multiplexer. ACS units update path metrics and send 2 decision bits to SMU for each update.

SMU adopts a classical 2-pointer traceback scheme [3] which allows 2 read pointer and 1 write pointer to access the memory at one time. The memory is divided into 4 banks. The depth of each bank is 16 which is approximately 5~6 times of constraint length. Each state requires 2 decision bits for QPSK. Therefore, the memory width is 128 bits for a total of 64 states. The trace-back latency of the SMU is 64 clock cycles.

The proposed ML detector has been implemented on an Altera Stratix II EP2S180 FPGA. We employ a fully parallel strategy for BMU and ACS in order to achieve a high throughput rate. The resource usage is shown in Table 1. At the system clock rate of 50 MHz, the corresponding throughput is 100 Mb/s. It is observed that the BMU becomes a crucial restriction on implementations since it consumes a significant number of multipliers, i.e. up to 67% DSP blocks of the target FPGA device.

We analyze the implementation complexity of the ML detector under general scenarios. Consider a cooperative communication system using M-ary modulation, where the half-duplex period for the relay is p. The estimated implementation cost is shown in Table 2, where a fully parallel implementation is assumed. The quantization precision for the input symbol and the path metric are q_1-bit and q_2-bit, respectively. The cost of the whitening filter is not considered here since it is negligible compared with that of the MLSE. The memory estimation for SMU is based on the 2-pointer scheme which can be carried out with 4 memory banks. The width of memory is $\log_2 M \cdot M^{L_{eff}-1}$ bits, and the depth of each memory bank is L_{tb} which is empirically set to $5 \sim 6(L_{eff} - 1)$.

Similar to the traditional MLSE, the implentation cost of the proposed ML detector for cooperative communications increases exponentially with respect to the effective channel length. Furthermore, the overhead of the proposed detector comprises the whitening filter, the multiplexers in BMU and additional control logic for phase synchronization, compared with the traditional MLSE.

Therefore, it is important to note that the proposed ML detector is optimal in ISI channels, however, when channels become even moderate lengths as expected in practice, or when the relay cooperation period is long, the implementation of the optimal detector becomes impractical.

Table 2: Implementation cost estimation of the proposed detector

	BMU	ACS	SMU	Total
Adder	$3M^{L_{eff}}$	$M^{L_{eff}}$	—	$4M^{L_{eff}}$
Multiplier	$2M^{L_{eff}}$	—	—	$2M^{L_{eff}}$
$2p$-to-1 multiplexer	$2q_1M^{L_{eff}}$	—	—	$2q_1M^{L_{eff}}$
M-to-1 multiplexer	—	$2q_2M^{L_{eff}-1}$	—	$2q_2M^{L_{eff}-1}$
Comparator	—	$(M-1)M^{L_{eff}-1}$	—	$(M-1)M^{L_{eff}-1}$
Memory (bits)	—	—	$4L_{tb}\log_2 M \cdot M^{L_{eff}-1}$	$4L_{tb}\log_2 M \cdot M^{L_{eff}-1}$

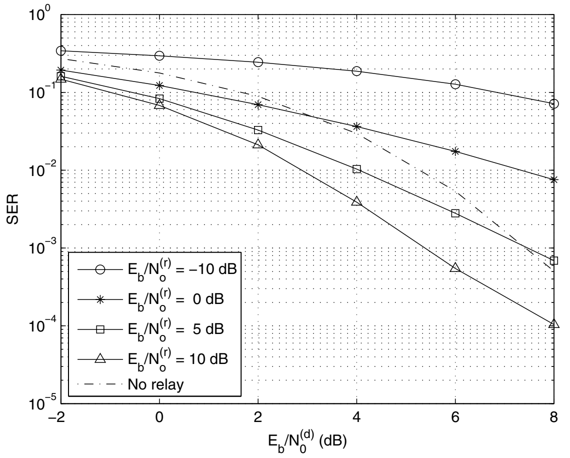

Figure 4: SER performance comparison with and without relay for a variety of source-relay bit-energy-to-noise ratios

5. NUMERICAL RESULTS

We simulate the symbol error rate (SER) performance of the proposed ML detector with the following parameters: the transmitted QPSK signal is i.i.d unit-power symbols $x[n] = \{\pm 1, \pm i\}$, the channels are $\boldsymbol{h}_{sd} = [0.3, -0.15, 0.9 + 0.3i]^T, \boldsymbol{h}_{sr} = [0.58, 0.58 + 0.58i]^T, \boldsymbol{h}_{rd} = [0.41, 0.82 - 0.41i]^T$ with unit channel gain $\|\boldsymbol{h}_{\{sd,sr,rd\}}\|^2 = 1$, the relay parameters are $\beta = 1$, $p = 1$. $E_b/N_o^{(d)}$ and $E_b/N_o^{(r)}$ denote bit-energy-to-noise ratio for destination and relay, respectively.

Figure 4 demonstrates the SER performance with respect to $E_b/N_o^{(d)}$ under different $E_b/N_o^{(r)}$. It is observed that as $E_b/N_o^{(r)}$ increases, the SER performance improves accordingly, which is due to the superior SNR in the source-relay-destination link. Additionally, the SER performance of non-relay system is also provided as a useful benchmark to evaluate the performance gain from the relay. Simulations demonstrate that the performance of a relay system in static ISI channels is not always superior to that of non-relay systems; that is, the relay does not typically result in a performance gain when the source-relay channel has relatively lower SNR. It suggests that whether or not the relay could help depends on the source-relay-destination link. In practice, it is necessary to shut down the relay when it does not improve the information reception at the receiver end.

6. CONCLUSION

In this paper, we built the system model with an AF half-duplex relay for cooperative communications in ISI channels. The periodically time varying property of the effective channel is investigated. An optimal ML detector consisting of the whitening filter and the modified Viterbi decoder is proposed and implemented on an FPGA. The complexity of the optimal detector increases exponentially with respect to the effective channel length. When compared to the traditional Viterbi detector, the detector requires added complexity in the form of a whitening filter, additional multiplexers, and control logic for phase synchronization. Numerical simulations of the proposed detector demonstrate that the SER performance improvement by adding a relay depends on the quality of source-relay-destination link. Future work will investigate a suboptimal detector which is feasible for large effective channel lengths.

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