Age of Information in Energy Harvesting Status Update Systems: When to Preempt in Service?

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***Abstract*—This paper analyzes the *age of information* in a single-source status update system with energy constraints. Specifically, a source provides status updates to a destination through a server assumed to have a battery with finite capacity that is replenished by harvesting energy. Arrival times of the status updates at the source and energy units at the server are assumed to be random according to independent Poisson processes, and service times are assumed to be exponentially distributed and independent of the status and energy arrivals. Average age expressions are derived under the assumption that new status updates from the source preempt a prior status update in service. Numerical results are provided to illustrate the operating regimes where a server that allows preemption of status update packets in service achieves a lower average age compared to a server that does not allow preemption.**

***Index Terms*—Age of information, average age, status update systems, energy harvesting, preemption, stochastic hybrid systems, Poisson point process.**

I. Introduction

Age of information (AoI) is a new measure of the timeliness/freshness of information in status update systems where the goal is to provide timely updates of a source's state to a destination. Examples of such systems analyzed include dissemination of channel state information in wireless communications, distributed sensor networks, and intelligent transportation. Early work on AoI showed the existence of an optimal AoI-minimizing server utilization which differed from strategies that maximized throughput or minimized delay [1]. This somewhat surprising result led to the study of AoI in a variety of contexts with different constraints, e.g., [2]–[21].

This paper analyzes AoI in a single-source status update system assuming the server has finite battery capacity and replenishes its battery by harvesting energy. A handful of recent papers [3], [4], [17]–[21] have also considered AoI under energy constraints. The focus of this prior work has primarily been on optimizing the schedule of status updates to minimize AoI. Specifically, [3], [4], [17], [19], [20] all assume the source can generate status updates at will. Much of this prior work has also focused on the infinite battery regime [3], [4], [17], [18]. Recent work focusing on the finite-battery setting [19], [20] assumes service times are negligible.

In this paper, we take a different approach by analyzing AoI as a function of the status update and energy arrival rates, the service rate, and the battery capacity assuming that new status updates from the source preempt prior status updates in service. A similar analysis was conducted in [21] for the case where the server did not allow preemption. Extending this work to consider preemption is motivated by [2] where it was shown that preemption can improve AoI in status update systems without energy constraints. In this paper, we show that preemption can also improve AoI for status update systems with energy constraints, but only in certain operating regimes. In addition to the closed-form expressions for the average age with preemption, numerical results are provided to illustrate the operating regimes where a server that allows preemption of status update packets in service achieves a lower average age compared to a server that does not allow preemption of packets in service.

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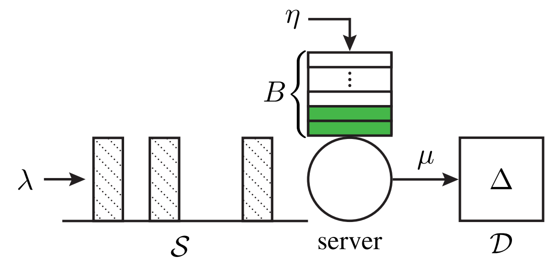

Fig. 1. The single-source status update system with an energy harvesting server with a battery capacity of B. Status updates arrive at the source with rate λ, energy units arrive at the server with rate η, and packets in service depart the server (complete service) with rate μ.

II. System Model

A. Status Update System

We consider a system with one source node $\mathcal{S}$ and one destination node $\mathcal{D}$ as represented in Fig. 1. In the absence of energy constraints at the server, this system model is identical to the single-source M/M/1 system with last-come-first-served discipline and preemption in service in [8]. The source intends to share information about its *time-varying state* with the destination and generates packets containing status updates at successive times based on a Poisson (point) process with rate λ. The source is assumed to send its packets to the destination through a server with service rate μ. The server is assumed to use energy from a finite capacity battery to service packets from the source. As in [3], [19]–[21], we assume that energy units are discrete and normalized so that energy arrivals always correspond to one unit of energy and the service of a packet consumes one unit of energy upon completion of service. The battery is replenished through a random energy harvesting process such that energy units arrive at the server according to a Poisson (point) process with rate η. The server's battery capacity is denoted as B units of energy. The random processes associated with the arrivals of energy units, status update packets, and service times are all assumed to be independent.

Packets containing status updates from the source immediately enter service when the server is idle and the battery is not empty. Given that there is more than one energy unit in the battery, packets that arrive when the server is busy preempt the packet in service. In this case, the packet in service is immediately dropped, the new arriving packet enters service, and the battery level reduces by one energy unit. Observe that here two events cause consumption of an energy unit: (i) service completion of a packet, and (ii) preemption of a packet in service. Arriving energy units at the server are stored in the battery only if the battery is not full at the time of arrival. We employ the same system model as in [21], except that the server is allowed to preempt a packet in service as in [5], [8].

For notational convenience, we define the normalized rates

$$\rho \triangleq \frac{\lambda}{\mu}, \qquad \beta \triangleq \frac{\eta}{\mu}, \tag{1}$$

where ρ represents the *server utilization* [1] and β represents the *energy utilization*, i.e., the rate at which the energy units arrive at the server normalized by the service rate.

B. Average Age Metric

Figure 2 shows an example age $\Delta(t)$ of the state information of the source from the perspective of the destination and the state of the battery $b(t)$ over time. The age $\Delta(t)$ is a linearly increasing random process when no updates arrive at the destination, and it has downward jumps when an update finishes service. Without loss of generality, assume that the observation begins at $t = 0$ when the queue is empty and the age is $\Delta(0)$. The j^{th} update of the source that was generated at time t_j departs the system at time t'_j. Here, the updates that get preempted during service are not indexed. Also, when a packet in service gets preempted by a new packet, the energy unit that was assigned to transmit the original packet is lost. At time t'_j, the age of the state information of the source from the perspective of the destination is reset to $T_j \triangleq t'_j - t_j$, which forms the sawtooth pattern because of the linear growth of age over time. The average age of the status updates of the source at the destination is equal to the area under $\Delta(t)$ divided by the observation interval. In general over an observation interval $(0, \mathcal{T})$, the average age is defined as

$$\Delta_{\mathcal{T}} \triangleq \frac{1}{\mathcal{T}} \int_0^{\mathcal{T}} \Delta(t)\ dt. \tag{2}$$

Letting the observation interval become large, the average age of the state information of the source from the perspective of the destination is [1]

$$\Delta \triangleq \lim_{\mathcal{T}\to\infty} \Delta_{\mathcal{T}}. \tag{3}$$

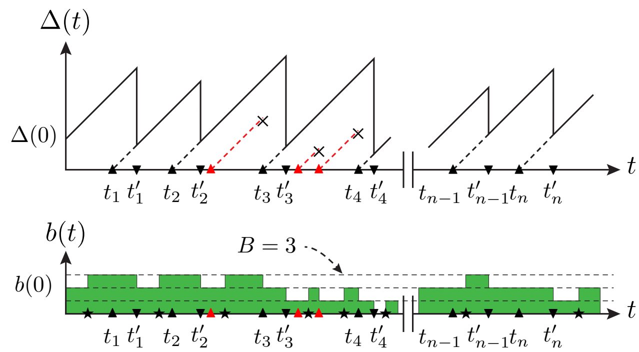

Fig. 2. An example evolution of the age $\Delta(t)$ and the battery state $b(t)$. Arrival times of the packets that get delivered to the destination are marked by ▲, departure times of these packets are marked by ▼, and arrival times of the packets that get preempted are marked by (red) ▲. We assume at time $t = 0$ the battery has $b(0) = 2$ units of energy stored in it, $B = 3$ and arrival times of the energy units are marked by ★.

III. Average Age Analysis

Table I represents a summary of the four possible cases for servers either unable or able to harvest energy while a packet is in service, and also whether preemption of packets in service is allowed or not. It was previously shown in [21] that when the server is able to harvest energy while a packet is in service, a lower average age is achieved compared to the case that the server is unable to harvest energy while a packet is in service. In the following we consider the single-source status update system where preemption of packets in service is allowed. We derive closed-form expressions for the average age Δ. To compute the average age, we use the SHS approach that was first used in [8] to evaluate the average age in the context of AoI. The SHS method defines a discrete state $q(t) \in \mathcal{Q}$ determining the state of the system with respect to the packets in service and the energy units in the battery. Associated with the SHS method is a continuous state $\mathbf{x}(t)$ that keeps track of the age over time. When a transition $q \to q'$ between two states occurs, the continuous state can have discontinuous jumps $\mathbf{x}(t^-) \to \mathbf{x}'(t) = \mathbf{x}(t^+)$. For a detailed description of the SHS method the reader is referred to [22].

TABLE I
DIFFERENT CASES WITH RESPECT TO ENERGY HARVESTING AND PREEMPTION OF THE PACKET IN SERVICE.

	w/o EH during service	w/ EH during service
w/o preemption	case A [21]	case B [21]
w/ preemption	case C	case D

Case C: Server Unable to Harvest Energy While Packet in Service, Preemption of a Packet in Service Allowed

A Markov chain representation of state $q(t) \in \mathcal{Q}$ of the system is shown in Fig. 3. The states are indexed by $q \in \mathcal{Q} = \{0, 1, 2, \ldots, 2B\}$. Each state is also associated with an (i, j) tuple where $i \in \{0, \ldots, B\}$ represents the

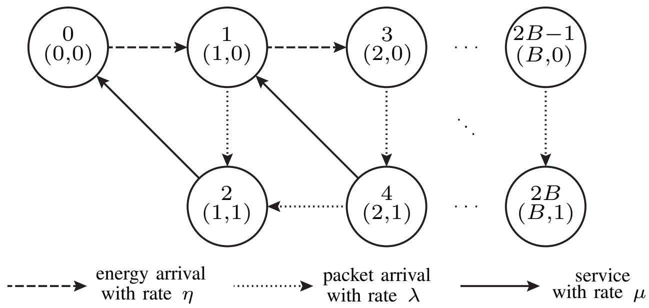

Fig. 3. The Markov chain representation of the single-source status update system with an energy harvesting server and battery capacity of $B \geq 1$ units of energy, where the server cannot harvest energy during service and preemption of packets in service is allowed. A dashed line, a dotted line and a solid line represent the arrival of an energy unit, the arrival of a packet from the source, and departure of the packet in service, respectively. In the (i, j) notation, i and j denote the number of energy units and status updates in the system, respectively. States are indexed by $q \in \mathcal{Q} = \{0, 1, \ldots, 2B\}$.

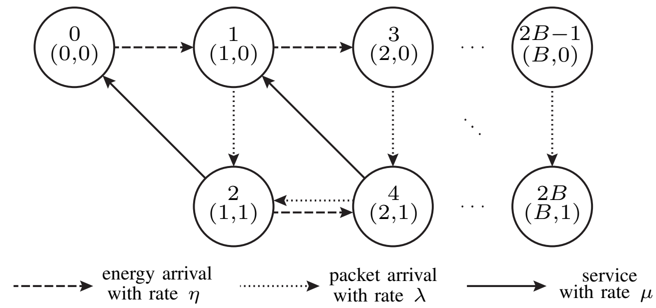

Fig. 4. The Markov chain representation of the single-source status update system with an energy harvesting server and battery capacity of $B \geq 1$ units of energy, where the server can harvest energy during service and preemption of packets in service is allowed. A dashed line, a dotted line, and a solid line represent the arrival of an energy unit, the arrival of a packet from the source, and departure of the packet in service, respectively. In the (i, j) notation, i and j denote the number of energy units and status updates in the system, respectively. States are indexed by $q \in \mathcal{Q} = \{0, 1, \ldots, 2B\}$.

number of energy units in the server's battery and $j \in \{0, 1\}$ represents the number of packets in service. Table II represents the exponential rates between the states in the Markov chain in Fig. 3 and the transition maps for each link ℓ. While the SHS analysis becomes intractable for general B, we use the SHS method to derive closed-form average age expressions for $B \in \{1, 2\}$ and we also derive asymptotic results for all B. The SHS method is also used to efficiently compute numerical results in Section IV.

TABLE II
TRANSITION RATES FOR THE MARKOV CHAIN IN FIG. 3, $2 \leq k \leq B$.

link ℓ	$q \to q'$	rate	$\phi(q, x) = (q', x')$
0	$0 \to 1$	$\eta\delta_{0,q}$	$(1, [x_0, 0])$
1	$1 \to 2$	$\lambda\delta_{1,q}$	$(2, [x_0, x_0])$
2	$2 \to 0$	$\mu\delta_{2,q}$	$(0, [x_0 - x_1, 0])$
$3k-3$	$2k-3 \to 2k-1$	$\eta\delta_{2k-3,q}$	$(2k-1, [x_0, 0])$
$3k-2$	$2k-1 \to 2k$	$\lambda\delta_{2k-1,q}$	$(2k, [x_0, x_0])$
$3k-1$	$2k \to 2k-3$	$\mu\delta_{2k,q}$	$(2k-3, [x_0 - x_1, 0])$
$3k$	$2k \to 2k-2$	$\lambda\delta_{2k,q}$	$(2k-2, [x_0, x_0])$

For $B = 1$, since the Markov chain is identical to the Markov chains of cases A and B, the average age is the same as Theorem 1 in [21]. For $B = 2$ the average age can be written as $\Delta_{\mathsf{C}} = X/Y$ where

$$\begin{aligned} X &\triangleq \beta^3(2\rho^3+3\rho^2+3\rho+1)+\beta^2(2\rho^4+6\rho^3+4\rho^2+\rho) \\ &\quad +\beta(2\rho^4+4\rho^3+\rho^2)+\rho^4+\rho^3, \\ Y &\triangleq \mu[\beta^3(\rho^3+2\rho^2+\rho)+\beta^2(\rho^4+3\rho^3+\rho^2)+\beta(\rho^4+\rho^3)]. \end{aligned}$$

For general B, the average age can be written as

$$\Delta_{\mathsf{C}} = \frac{\beta^{B+1} f_0(\rho) + \beta^B f_1(\rho) + \ldots + \beta f_B(\rho) + f_{B+1}(\rho)}{\mu[\beta^{B+1} g_0(\rho) + \beta^B g_1(\rho) + \ldots + \beta g_B(\rho)]}, \quad (4)$$

where $f_i(\rho)$ and $g_j(\rho)$ are polynomial functions of ρ with degree of at most $2B$ for $i \in \{0,1,\ldots,B+1\}$ and $j \in \{0,1,\ldots,B\}$.

The form of (4) allows us to derive an asymptotic result for the case when the energy arrival rate is large. For fixed λ, μ, and B, when $\eta \to \infty$ or, equivalently, $\beta \to \infty$, from (4) it can be shown that

$$\lim_{\beta \to \infty} \Delta_{\mathsf{C}} = \begin{cases} \dfrac{2\rho^2 + 2\rho + 1}{\mu(\rho^2 + \rho)} & B = 1 \\ \dfrac{f_0(\rho)}{\mu g_0(\rho)} & B \geq 2 \end{cases}, \quad (5)$$

where

$$f_0(\rho) = (\rho + 1)^{B+1} + \rho^{B+1}, \quad g_0(\rho) = \rho(\rho + 1)^B.$$

For fixed η, μ, and B, when the status update arrival rate becomes large, i.e., $\lambda \to \infty$ or, equivalently, $\rho \to \infty$, the average age of this case is identical to case A in [21]. Intuitively, this follows from the fact that the battery is always either empty or has one energy unit when $\rho \to \infty$. In the steady state, the system is always traversing the states at the leftmost end of the Markov chain, i.e., states 0, 1, and 2, and there is no advantage in being able to preempt the packet in service. Similarly, for fixed λ, η, and B, when the service rate becomes large, the average age of this case is identical to cases A and B where preemption of packets in service is not allowed. Intuitively, when $\mu \to \infty$, as soon as a packet is presented to the server, it is instantaneously delivered. Hence, the probability of a status update arriving during service becomes small and the Markov chain becomes identical to the Markov chains of cases A and B in [21].

Case D: Server Able to Harvest Energy While Packet in Service, Preemption of a Packet in Service Allowed

A Markov chain representation of state $q(t) \in \mathcal{Q}$ of the system is shown in Fig. 4. The states are indexed by $q \in \mathcal{Q} = \{0, 1, 2, \ldots, 2B\}$. Theorem 1 provides an expression for the average age of this status update system.

Theorem 1. *For $B \geq 2$ the average age of the status update system where the server is able to harvest energy while a* *packet is in service with preemption in service allowed is*

$$\Delta_{\mathsf{D}} = \begin{cases} \dfrac{B\rho^2 + (2B+2)\rho + B + 2}{\mu[B\rho^2 + (B+1)\rho]} & \beta = \rho \\ \dfrac{(\rho+1)^2\beta^{B+2} - (\beta+1)^2\rho^{B+2}}{\mu[(\rho^2+\rho)\beta^{B+2} - (\beta^2+\beta)\rho^{B+2}]} & \beta \neq \rho \end{cases}. \quad (6)$$

Note that for $B = 1$, the system is identical to the case in Theorem 1 in [21] and so is the average age. From the SHS method, the continuous state vector is denoted by $\mathbf{x}(t) = [x_0(t), x_1(t)]$, where $x_0(t)$ represents the current age $\Delta(t)$ and $x_1(t)$ represents the reduction in $\Delta(t)$ that will occur when the packet in service is delivered. Table III represents the

TABLE III
TRANSITION RATES FOR THE MARKOV CHAIN IN FIG. 4, $2 \leq k \leq B-1$.

link ℓ	$q \to q'$	rate	$\phi(q,x) = (q',x')$
0	$0 \to 1$	$\eta\delta_{0,q}$	$(1, [x_0, 0])$
1	$1 \to 2$	$\lambda\delta_{1,q}$	$(2, [x_0, x_0])$
2	$1 \to 3$	$\eta\delta_{1,q}$	$(3, [x_0, 0])$
3	$2 \to 0$	$\mu\delta_{2,q}$	$(0, [x_0 - x_1, 0])$
4	$2 \to 4$	$\eta\delta_{2,q}$	$(4, [x_0, x_1])$
$5k-5$	$2k-1 \to 2k$	$\lambda\delta_{2k-1,q}$	$(2k, [x_0, x_0])$
$5k-4$	$2k-1 \to 2k+1$	$\eta\delta_{2k-1,q}$	$(2k+1, [x_0, 0])$
$5k-3$	$2k \to 2k-3$	$\mu\delta_{2k,q}$	$(2k-3, [x_0 - x_1, 0])$
$5k-2$	$2k \to 2k-2$	$\lambda\delta_{2k,q}$	$(2k-2, [x_0, x_0])$
$5k-1$	$2k \to 2k+2$	$\eta\delta_{2k,q}$	$(2k+2, [x_0, x_1])$
$5B-5$	$2B-1 \to 2B$	$\lambda\delta_{2B-1,q}$	$(2B, [x_0, x_0])$
$5B-4$	$2B \to 2B-3$	$\mu\delta_{2B,q}$	$(2B-3, [x_0 - x_1, 0])$
$5B-3$	$2B \to 2B-2$	$\lambda\delta_{2B,q}$	$(2B-2, [x_0, x_0])$

exponential rates at which state $q(t^-)$ transitions to $q'(t) = q(t^+)$ in the Markov chain in Fig. 4 and the transition map $\phi(q(t^-), \mathbf{x}(t^-)) = (q'(t), \mathbf{x}'(t)) = (q(t^+), \mathbf{x}(t^+))$ for each link ℓ. Due to space constraints, we provide a sketch and omit detailed derivations in the following. The SHS method defines test functions whose expected values converge to steady-state quantities of interest like the average age. By applying an extended generator function and some conditions to the test functions, a set of first order linear differential equations are obtained. By solving the differential equations and some algebra, the average age in (6) is obtained.

Similar to Theorem 1 in [21] note that (6) is symmetric with respect to the $\beta = \rho$ line. In other words, for any μ and B, average age is invariant to exchanging β and ρ. This is somewhat surprising since packets and energy are handled differently by the server. Specifically, up to B units of energy can be stored by the server, whereas only one packet can be in service at any time. Comparing the average age expressions for cases A and D, we can write

$$\Delta_{\mathsf{A}} - \Delta_{\mathsf{D}} = \begin{cases} 0 & B = 1 \\ \dfrac{\rho^2\beta^2 \displaystyle\sum_{k=0}^{B-1} \beta^k \rho^{B-k-1}}{\mu\rho\beta\left[(\rho+1)\displaystyle\sum_{k=1}^{B} \beta^k \rho^{B-k} + \rho^B\right]} & B \geq 2 \end{cases},$$

which shows $\Delta_{\mathsf{A}} - \Delta_{\mathsf{D}} \geq 0$ regardless of the system parameters ρ, β, μ, and B. The remainder of this section considers asymptotic results. First, fixing η, μ, and B, when the status update arrival rate becomes large, i.e., $\lambda \to \infty$ (or $\rho \to \infty$), we can write

$$\lim_{\rho\to\infty} \Delta_{\mathsf{D}} = \frac{1}{\mu}\left(1 + \frac{1}{\beta}\right). \quad (7)$$

Second, for fixed λ, μ, and B, when the energy arrival rate becomes large, i.e., $\eta \to \infty$ (or $\beta \to \infty$), we can write

$$\lim_{\beta\to\infty} \Delta_{\mathsf{D}} = \frac{1}{\mu}\left(1 + \frac{1}{\rho}\right), \quad (8)$$

which is identical to the average age expression of the single-source M/M/1 status update system with last-come-first-served discipline and preemption in service in [8], and also the average age of the best-effort updating policy in [3].

Third, for fixed λ, η, and B, when the service rate becomes large, we can write

$$\lim_{\mu\to\infty} \Delta_{\mathsf{D}} = \begin{cases} \dfrac{B+2}{(B+1)\lambda} & \beta = \rho \\ \dfrac{\eta^{B+2} - \lambda^{B+2}}{\lambda\eta^{B+2} - \eta\lambda^{B+2}} & \beta \neq \rho. \end{cases} \quad (9)$$

Observe that when $\mu \to \infty$, all of the four cases A, B, C, and D achieve the same average age.

Finally, for fixed λ, η, and μ, when the battery becomes large, we can write

$$\lim_{B\to\infty} \Delta_{\mathsf{D}} = \begin{cases} \dfrac{1}{\mu}\left(1 + \dfrac{1}{\beta}\right) & \beta < \rho \\ \dfrac{1}{\mu}\left(1 + \dfrac{1}{\rho}\right) & \beta \geq \rho. \end{cases} \quad (10)$$

Also, when $\beta \to 0$, $\rho \to 0$, or $\mu \to 0$, we have $\lim_{\beta\to 0} \Delta_{\mathsf{D}} = \infty$, $\lim_{\rho\to 0} \Delta_{\mathsf{D}} = \infty$, and $\lim_{\mu\to 0} \Delta_{\mathsf{D}} = \infty$, respectively.

IV. NUMERICAL RESULTS

This section provides numerical examples to illustrate the operating regimes where preemption of status updates in service achieves a lower average age compared to a server that does not allow preemption of packets in service [21]. Figure 5 plots the ratio $\Delta_{\mathsf{C}}/\Delta_{\mathsf{A}}$, where the server is unable to harvest energy while a packet is in service, and the ratio $\Delta_{\mathsf{D}}/\Delta_{\mathsf{B}}$, where the server is able to harvest energy while a packet is in service. Regions shaded in green and yellow correspond to improved or degraded performance, respectively, with preemption.

In the yellow shaded areas, preemption results in degraded performance. Intuitively, the yellow shaded area corresponds to an "energy starved" operating regime, where energy arrivals are relatively infrequent with respect to the rate of information arrivals and the service rate. Preemption of packets in service leads to faster depletion of the battery which increases the probability of new status updates being dropped because of an empty battery. This in turn leads to less frequent delivery of status updates to the destination and degraded performance.

In the green shaded areas, preemption improves performance. Intuitively, the green shaded area corresponds to an "energy rich" operating regime. In this regime, the probability

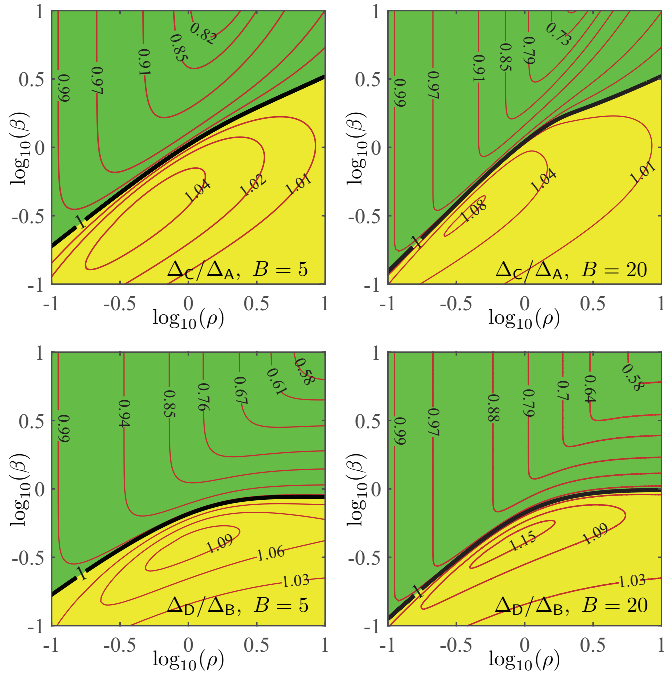

Fig. 5. Contours of the ratio of the average age of cases with or without preemption, for $0.1 \le \beta \le 10$, $0.1 \le \rho \le 10$, $\mu = 1$, and $B = \{5, 20\}$. The green and yellow regions represent the areas where the ratio is less than one, and the ratio is greater than one, respectively.

of the battery becoming depleted is small and the effect of the energy constraint is less significant. Similar to [2], which analyzed the average age of a server that allowed preemption without energy constraints, we see that the performance achieved by preemption is better than the average age achieved by a server that does not allow preemption of packets in service in the "energy rich" regime. We also observe that the average age reduction due to preemption becomes more considerable when the server is able to harvest energy while a packet is in service compared to cases where the server is unable to harvest energy while a packet is in service. Intuitively, this is because the server's ability to harvest energy while a packet is in service helps avoid wasting the arriving energy units as long as the battery in not full.

In general preemption of the packets in service is not the best packet management policy all the times, and the best policy depends on the system parameters and the servers ability to harvest energy while a packet is in service.

V. Conclusions

This paper studied the AoI problem in a single-source status update system with an energy harvesting server with finite battery capacity. For servers able to harvest energy during service and with preemption of packets in service allowed, the average age expression was derived as a function of the system parameters. Numerical results were provided to illustrate the operating regimes where preemption of status updates in service achieves a lower average age compared to a server that does not allow preemption of packets in service.

An interesting direction for future work is to preempt the packets in service with some probability and adaptively optimizing this probability to minimize the average age.

References

[1] S. Kaul, R. Yates, and M. Gruteser, "Real-time status: How often should one update?" in *Proc. IEEE INFOCOM*, Mar. 2012, pp. 2731–2735.

[2] S. K. Kaul, R. D. Yates, and M. Gruteser, "Status updates through queues," in *Proc. Conference on Inf. Sciences and Systems (CISS)*, Mar. 2012, pp. 1–6.

[3] R. D. Yates, "Lazy is timely: Status updates by an energy harvesting source," in *Proc. IEEE Intl. Symp on Inf. Theory (ISIT)*, Jun. 2015, pp. 3008–3012.

[4] B. T. Bacinoglu, E. T. Ceran, and E. Uysal-Biyikoglu, "Age of information under energy replenishment constraints," in *Proc. Inf. Theory and Applications Workshop (ITA)*, Feb. 2015, pp. 25–31.

[5] M. Costa, M. Codreanu, and A. Ephremides, "On the age of information in status update systems with packet management," *IEEE Trans. Inf. Theory*, vol. 62, no. 4, pp. 1897–1910, Feb. 2016.

[6] S. Farazi, A. G. Klein, and D. R. Brown III, "On the average staleness of global channel state information in wireless networks with random transmit node selection," in *Proc. IEEE Intl. Conf. on Acoustics, Speech and Signal Processing (ICASSP)*, Mar. 2016, pp. 3621–3625.

[7] S. Farazi, D. R. Brown III, and A. G. Klein, "On global channel state estimation and dissemination in ring networks," in *Proc. Asilomar Conf. on Signals, Systems, and Computers*, Nov. 2016, pp. 1122–1127.

[8] R. D. Yates and S. K. Kaul, "The age of information: Real-time status updating by multiple sources," *arXiv preprint arXiv:1608.08622*, 2016.

[9] C. Kam, S. Kompella, G. D. Nguyen, J. E. Wieselthier, and A. Ephremides, "Age of information with a packet deadline," in *Proc. IEEE Intl. Symp. on Information Theory (ISIT)*, Jul. 2016, pp. 2564–2568.

[10] K. Chen and L. Huang, "Age-of-information in the presence of error," in *Proc. IEEE Intl. Symp on Inf. Theory (ISIT)*, Jul. 2016, pp. 2579–2583.

[11] Y. Sun, E. Uysal-Biyikoglu, R. D. Yates, C. E. Koksal, and N. B. Shroff, "Update or wait: How to keep your data fresh," *IEEE Trans. Inf. Theory*, vol. 63, no. 11, pp. 7492–7508, Aug. 2017.

[12] A. Kosta, N. Pappas, A. Ephremides, and V. Angelakis, "Age and value of information: Non-linear age case," in *Proc. IEEE Intl. Symp on Inf. Theory (ISIT)*, Aug. 2017, pp. 326–330.

[13] Q. He, D. Yuan, and A. Ephremides, "Optimal link scheduling for age minimization in wireless systems," *IEEE Trans. Inf. Theory*, 2017.

[14] R. D. Yates, P. Ciblat, A. Yener, and M. Wigger, "Age-optimal constrained cache updating," in *Proc. IEEE Intl. Symp on Inf. Theory (ISIT)*, Aug. 2017, pp. 141–145.

[15] A. G. Klein, S. Farazi, W. He, and D. R. Brown III, "Staleness bounds and efficient protocols for dissemination of global channel state information," *IEEE Trans. Wireless Commun.*, vol. 16, no. 9, pp. 5732–5746, Sep. 2017.

[16] S. Farazi, A. G. Klein, and D. R. Brown III, "Bounds on the age of information for global channel state dissemination in fully-connected networks," in *Proc. Intl. Conf. on Computer Communication and Networks (ICCCN)*, Jul. 2017, pp. 1–7.

[17] A. Arafa and S. Ulukus, "Age-minimal transmission in energy harvesting two-hop networks," in *Proc. IEEE Global Telecomm. Conf. (GLOBECOM)*, Dec. 2017.

[18] ——, "Age minimization in energy harvesting communications: Energy-controlled delays," in *Proc. Asilomar Conf. on Signals, Systems, and Computers*, Oct. 2017.

[19] X. Wu, J. Yang, and J. Wu, "Optimal status updating to minimize age of information with an energy harvesting source," in *Proc. IEEE Intl. Conf. on Comm. (ICC)*, Jul. 2017, pp. 1–6.

[20] B. T. Bacinoglu and E. Uysal-Biyikoglu, "Scheduling status updates to minimize age of information with an energy harvesting sensor," in *Proc. IEEE Intl. Symp. on Inf. Theory (ISIT)*, Aug. 2017, pp. 1122–1126.

[21] S. Farazi, A. G. Klein, and D. R. Brown III, "Average age of information for status update systems with an energy harvesting server," in *Proc. IEEE Intl. Conf. on Computer Comm. Workshops (INFOCOM WKSHPS)*, Apr. 2018.

[22] J. P. Hespanha, "Modelling and analysis of stochastic hybrid systems," *IEE Proceedings-Control Theory and Applications*, vol. 153, no. 5, pp. 520–535, 2006.