Average Age of Information for Status Update Systems with an Energy Harvesting Server

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***Abstract*—This paper investigates the *age of information* in a single-source status update system with energy constraints. Specifically, a source provides status updates to a destination through a server assumed to have a battery with finite capacity that is replenished by harvesting energy. Arrival times of the status updates at the source and energy units at the server are assumed to be random according to independent Poisson processes. Service times are also assumed to be exponentially distributed and independent of the status and energy arrivals. The server is assumed to always be able to harvest energy when no information packet from the source is in service and the battery is not full. With regards to harvesting energy while packets are in service, this paper analyzes the average age of information both for servers that are able or unable to perform simultaneous service and energy harvesting. Closed-form expressions for the average age in terms of the system parameters are derived. Simulation results confirm the analysis and numerically demonstrate the performance advantage of servers with simultaneous service and energy harvesting.**

***Index Terms*—Age of information, average age, status update systems, energy harvesting, stochastic hybrid systems.**

I. Introduction

In recent years there has been a growing interest in real-time state update communication systems, where a source is required to provide *timely* updates of some time-varying state to a destination. Examples of these scenarios include dissemination of channel state information in wireless communication systems, distributed sensor networks, consensus algorithms, environmental temperature monitoring, and intelligent transportation. Since timely updates are critical in these and many other applications, an important problem is to understand the freshness of information in status update systems.

To evaluate the freshness of information in status update systems, an *age of information* (AoI) metric was proposed in [1] for sharing timely status updates. This seminal work led to several additional studies including work on the AoI of single-source status update systems, e.g., [2]–[14], and papers on the AoI of multi-source status update systems [15]–[28].

While there has been considerable work on studying AoI in status update systems without energy constraints, only a handful of recent papers [29]–[34] have considered AoI in status update systems with energy constraints. This prior work has mainly focused on optimizing the schedule of status updates from the source to minimize the AoI in different scenarios subject to energy constraints. Specifically, [29]–[31], [33], [34] all assume the source can generate status updates at any time. The goal is to optimize the timing of the status updates from the source to minimize the average age in various settings with energy constraints. Much of this work has focused on the infinite battery regime [29]–[32]. Recent work focusing on the finite-battery setting [33], [34] assumes always-available source updates and instantaneous service.

While this paper also considers average age of status update systems under energy constraints, we take a somewhat different approach than the previous work in this area. First, we assume that status updates from the source are not always available, but instead arrive at the server at random times. Similarly, energy arrivals and service times are also assumed to be random. Since we do not control the timing of the status updates, we consider a simple, fixed, real-time update policy where new source updates enter service if the server is idle and has sufficient energy to service the packet. If the server is busy or does not have sufficient energy, the source update is dropped. Second, we use tools from stochastic hybrid systems (SHS) [35] to analyze the average age as a function of the arrival/service rates and the battery capacity for two different types of servers: servers unable to harvest energy while packets are in service and servers able to harvest energy while packets are in service. Third, our analysis considers both finite and infinite battery regimes without the simplifying assumptions of always-available source updates or instantaneous service. The main contribution of this paper is the derivation of closed-form expressions for average age in these settings. Our analysis reveals the effect of each system parameter on the average age: (i) status update arrival rate at the source λ, (ii) energy arrival rate at the server η, (iii) service rate μ, and (iv) server battery capacity B. Simulation results confirm the analysis and numerically demonstrate the performance advantage of servers able to harvest energy while servicing information packets from the source.

II. System Model

We consider a system with one source node $\mathcal{S}$ and one destination node $\mathcal{D}$ as represented in Fig. 1. In the absence of the energy constraints at the server, this system model is identical to the M/M/1/1 case in [3]. The source intends to share information about its *time-varying state* with the destination and generates packets containing status updates at successive times based on a Poisson (point) process with

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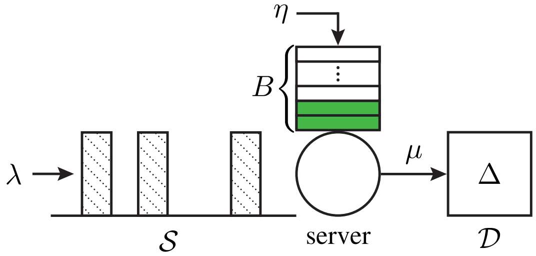

Fig. 1. The single-source status update system with an energy harvesting server with a battery capacity of B. Status updates arrive at the source with rate λ, energy units arrive at the server with rate η, and packets in service depart the server (complete service) with rate μ.

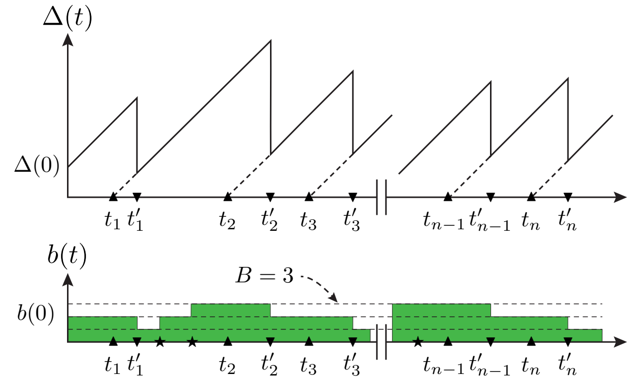

Fig. 2. An example evolution of the age $\Delta(t)$ and the battery state $b(t)$. Arrival times of the packets that get delivered to the destination are marked by ▲ and departure times of these packets are marked by ▼. We assume at time $t = 0$ the battery of capacity $B = 3$ has $b(0) = 2$ units of energy stored in it and arrival times of the energy units are marked by ★.

rate λ. The source is assumed to send its packets to the destination through a server with service rate μ. The server is assumed to use energy from a finite capacity battery to service packets from the source. As in [29], [33], [34], we assume that energy units are discrete and normalized so that energy arrivals always correspond to one unit of energy and the service of a packet consumes one unit of energy upon completion of service. The battery is replenished through a random energy harvesting process such that energy units arrive at the server according to a Poisson (point) process with rate η. The server's battery capacity is denoted as B units of energy. The random processes associated with the arrivals of energy units, status update packets, and service times are all assumed to be independent.

Packets containing status updates from the source immediately enter service only if (i) the server is idle and (ii) the server's battery is not empty. If either of these conditions are not satisfied when a packet is generated, the packet is ignored by the server and discarded. Packets in service are not preempted and no packets are held in a queue. Energy unit arrivals at the server are stored in the battery only if the battery is not full at the time of arrival.

For notational convenience, we define the normalized rates

$$\rho \triangleq \frac{\lambda}{\mu}, \qquad \beta \triangleq \frac{\eta}{\mu}, \tag{1}$$

where ρ represents the *server utilization* [1] and β represents the *energy utilization*, i.e., the rate at which the energy units arrive at the server normalized by the service rate.

A. Average Age Metric

Figure 2 shows an example age $\Delta(t)$ of the state information of the source from the perspective of the destination and the state of the battery $b(t)$ over time. The age $\Delta(t)$ is a linearly increasing random process when no updates arrive at the destination and has downward jumps when an update completes service. Without loss of generality, assume that the observation begins at $t = 0$ when the queue is empty and the age is $\Delta(0)$. The j^{th} update of the source generated at time t_j completes service and is delivered to the destination at time t'_j. At time t'_j, the age of the state information of the source from the perspective of the destination is reset to $T_j \triangleq t'_j - t_j$, which forms the sawtooth pattern because of the linear growth of age over time. The average age of the status updates of the source at the destination is equal to the area under $\Delta(t)$ divided by the observation interval. Over an observation interval $(0, \mathcal{T})$, the average age is defined as

$$\Delta_{\mathcal{T}} \triangleq \frac{1}{\mathcal{T}} \int_0^{\mathcal{T}} \Delta(t)\ dt. \tag{2}$$

Letting the observation interval become large, the average age of the state information of the source from the perspective of the destination is [1]

$$\Delta \triangleq \lim_{\mathcal{T}\to\infty} \Delta_{\mathcal{T}}. \tag{3}$$

III. Average Age Analysis

In this section we consider a single-source status update system with energy constraints for the server. We study two cases where (i) the server is unable to harvest energy while a packet is in service (case A), and (ii) the server can harvest energy while a packet is in service (case B), given the battery is not full. For case A we derive a closed-form expression for the average age Δ and compare it with the average age of case B, which is obtained numerically in Section IV. In the following, we refer to the average age expressions of cases A and B by Δ_{A} and Δ_{B}, respectively. To compute the average age, we use the SHS approach that was first used in [22] to evaluate the average age in the context of AoI. The SHS method defines a discrete state $q(t) \in \mathcal{Q}$ determining the state of the system with respect to the packets in service and the energy units in the battery. Associated with the SHS method is a continuous state $\mathbf{x}(t)$ that keeps track of the age over time. When a transition $q \to q'$ between two states occurs, the continuous state can have discontinuous jumps $\mathbf{x}(t^-) \to \mathbf{x}'(t) = \mathbf{x}(t^+)$. For more details on the SHS method the reader is referred to [35].

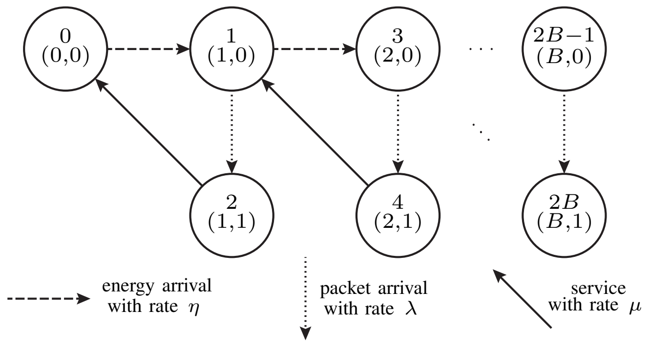

Fig. 3. The Markov chain representation of the single-source status update system with an energy harvesting server and battery capacity of $B > 1$ units of energy, where the server is unable to harvest energy during service. A dashed line, a dotted line, and a solid line represent the arrival of an energy unit, the arrival of a packet from the source, and departure of the packet in service, respectively. In the (i, j) notation, i and j denote the number of energy units and status updates in the system, respectively. States are indexed by $q \in \mathcal{Q} = \{0, 1, \ldots, 2B\}$.

Case A*: Server Unable to Harvest Energy While Packet in Service*

A Markov chain representation of state $q(t) \in \mathcal{Q}$ of the system is shown in Fig. 3. The states are indexed by $q \in \mathcal{Q} = \{0, 1, 2, \ldots, 2B\}$. Each state is also associated with an (i, j) tuple where $i \in \{0, \ldots, B\}$ represents the number of energy units in the server's battery and $j \in \{0, 1\}$ represents the number of packets in service. As seen in Fig. 3, when there is a packet in service, i.e., the system is in a positive even indexed state, no energy units are collected. Theorem 1 provides an expression for the average age of the status update system when the server is unable to harvest energy while a packet is in service.

Theorem 1. *The average age of the status update system where the server is unable to harvest energy while a packet is in service is*

$$\Delta_{\mathsf{A}} = \begin{cases} \dfrac{2B\rho^2 + (2B+2)\rho + B + 2}{\mu[B\rho^2 + (B+1)\rho]} & \beta = \rho \\ \dfrac{(2\rho^2+2\rho+1)\beta^{B+2} - (2\beta^2+2\beta+1)\rho^{B+2}}{\mu[(\rho^2+\rho)\beta^{B+2} - (\beta^2+\beta)\rho^{B+2}]} & \beta \neq \rho \end{cases}. \tag{4}$$

Due to space limitations, we only provide a sketch of the proof here. We follow essentially the same procedure as in [22], except here we have a larger state space (with $2B + 1$ states) as shown in Fig. 3. The SHS method defines test functions whose expected values converge to steady-state quantities of interest like the average age.

From the SHS method, let $\mathbf{x}(t) = [x_0(t), x_1(t)]$ denote the continuous state vector, where $x_0(t)$ represents the current age $\Delta(t)$ and $x_1(t)$ represents the reduction in $\Delta(t)$ that will occur when the packet in service is delivered. Table I represents the exponential rates at which state $q(t^-)$ transitions to $q'(t) = q(t^+)$ in the Markov chain in Fig. 3 and the transition map $\phi(q(t^-), \mathbf{x}(t^-)) = (q'(t), \mathbf{x}'(t)) = (q(t^+), \mathbf{x}(t^+))$ for each link ℓ. For example, the occurrence of link $\ell = 0$ at time t shows that the battery is empty and no packet is in service at time t^-; then, an energy unit arrives and the system transitions to state $q(t^+) = 1$. When this transition occurs, we have $x'_0(t) = x_0(t^+) = x_0(t^-) = x_0(t)$, and since there is no packet in service, $x'_1(t) = x_1(t^+) = 0$. By applying an extended generator function and some conditions to the test functions, a set of first order linear differential equations are obtained. By solving the differential equations, and after some additional algebra, an expression for the average age in (4) can be obtained.

TABLE I
TRANSITION RATES FOR THE MARKOV CHAIN IN FIG. 3, $2 \leq k \leq B$.

link ℓ	$q \to q'$	rate	$\phi(q, x) = (q', x')$
0	$0 \to 1$	$\eta\delta_{0,q}$	$(1, [x_0, 0])$
1	$1 \to 2$	$\lambda\delta_{1,q}$	$(2, [x_0, x_0])$
2	$2 \to 0$	$\mu\delta_{2,q}$	$(0, [x_0 - x_1, 0])$
$3k-3$	$2k-3 \to 2k-1$	$\eta\delta_{2k-3,q}$	$(2k-1, [x_0, 0])$
$3k-2$	$2k-1 \to 2k$	$\lambda\delta_{2k-1,q}$	$(2k, [x_0, x_0])$
$3k-1$	$2k \to 2k-3$	$\mu\delta_{2k,q}$	$(2k-3, [x_0 - x_1, 0])$

Note that (4) is symmetric with respect to the $\beta = \rho$ line. In other words, for any μ and B, average age is invariant to exchanging β and ρ. This is somewhat surprising since packets and energy are handled differently by the server. Specifically, up to B units of energy can be stored by the server, whereas only one packet can be in service at any time.

The remainder of this section considers asymptotic results. First, fixing η, μ, and B, when the status update arrival rate becomes large, i.e., $\lambda \to \infty$ or, equivalently, $\rho \to \infty$, we can write

$$\lim_{\rho\to\infty} \Delta_{\mathsf{A}} = \frac{2\beta^2 + 2\beta + 1}{\mu(\beta^2 + \beta)}. \tag{5}$$

Second, for fixed λ, μ, and B, when the energy arrival rate becomes large, i.e., $\eta \to \infty$ or, equivalently, $\beta \to \infty$, we can write

$$\lim_{\beta\to\infty} \Delta_{\mathsf{A}} = \frac{2\rho^2 + 2\rho + 1}{\mu(\rho^2 + \rho)}, \tag{6}$$

which is identical to the average age expression for the M/M/1/1 case in [3].

Third, for fixed λ, η, and B, when the service rate becomes large, we can write

$$\lim_{\mu\to\infty} \Delta_{\mathsf{A}} = \begin{cases} \dfrac{B+2}{(B+1)\lambda} & \beta = \rho \\ \dfrac{\eta^{B+2} - \lambda^{B+2}}{\lambda\eta^{B+2} - \eta\lambda^{B+2}} & \beta \neq \rho \end{cases}. \tag{7}$$

Finally, for fixed λ, η, and μ, when the battery becomes large, we can write

$$\lim_{B\to\infty} \Delta_{\mathsf{A}} = \begin{cases} \dfrac{2\beta^2 + 2\beta + 1}{\mu(\beta^2 + \beta)} & \beta < \rho \\ \dfrac{2\rho^2 + 2\rho + 1}{\mu(\rho^2 + \rho)} & \beta \geq \rho \end{cases}. \tag{8}$$

Also, when $\beta \to 0$, $\rho \to 0$, or $\mu \to 0$, we have $\lim_{\beta\to 0} \Delta_{\mathsf{A}} = \infty$, $\lim_{\rho\to 0} \Delta_{\mathsf{A}} = \infty$, and $\lim_{\mu\to 0} \Delta_{\mathsf{A}} = \infty$, respectively.

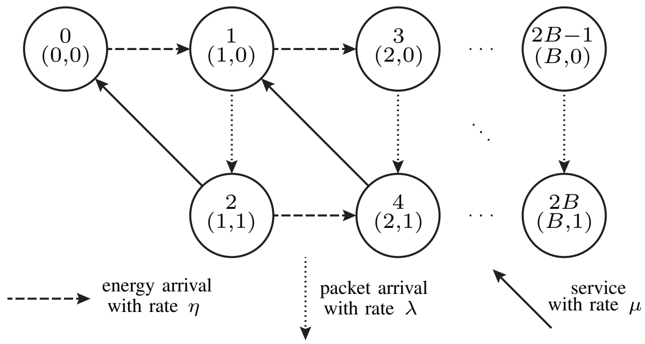

Fig. 4. The Markov chain representation of the single-source status update system with an energy harvesting server and battery capacity of $B > 1$ units of energy, where the server can harvest energy during service. A dashed line, a dotted line, and a solid line represent the arrival of an energy unit, the arrival of a packet from the source, and departure of the packet in service, respectively. In the (i, j) notation, i and j denote the number of energy units and status updates in the system, respectively. States are indexed by $q \in \mathcal{Q} = \{0, 1, \ldots, 2B\}$.

Case B*: Server Able to Harvest Energy While Packet in Service*

A Markov chain representation of state $q(t) \in \mathcal{Q}$ is shown in Fig. 4. Table II represents the exponential rates between the states in the Markov chain in Fig. 4 and the transition maps for each link ℓ. The difference between this model and the model for case A is that here, while a packet is in service and the battery is not full, an arriving energy unit is harvested. The additional links in Fig. 4 cause the SHS analysis to become intractable for general B, however. In this section, we use the SHS method to derive closed-form average age expressions for $B \in \{1, 2\}$ and also derive asymptotic results for all B. The SHS method is also used to efficiently compute numerical results in Section IV.

TABLE II
TRANSITION RATES FOR THE MARKOV CHAIN IN FIG. 4, $2 \le k \le B-1$.

link ℓ	$q \to q'$	rate	$\phi(q,x) = (q',x')$
0	$0 \to 1$	$\eta\delta_{0,q}$	$(1, [x_0, 0])$
1	$1 \to 2$	$\lambda\delta_{1,q}$	$(2, [x_0, x_0])$
2	$1 \to 3$	$\eta\delta_{1,q}$	$(3, [x_0, 0])$
3	$2 \to 0$	$\mu\delta_{2,q}$	$(0, [x_0 - x_1, 0])$
4	$2 \to 4$	$\eta\delta_{2,q}$	$(4, [x_0, x_1])$
$4k-3$	$2k-1 \to 2k$	$\lambda\delta_{2k-1,q}$	$(2k, [x_0, x_0])$
$4k-2$	$2k-1 \to 2k+1$	$\eta\delta_{2k-1,q}$	$(2k+1, [x_0, 0])$
$4k-1$	$2k \to 2k-3$	$\mu\delta_{2k,q}$	$(2k-3, [x_0 - x_1, 0])$
$4k$	$2k \to 2k+2$	$\eta\delta_{2k,q}$	$(2k+2, [x_0, x_1])$
$4B-3$	$2B-1 \to 2B$	$\lambda\delta_{2B-1,q}$	$(2B, [x_0, x_0])$
$4B-2$	$2B \to 2B-3$	$\mu\delta_{2B,q}$	$(2B-3, [x_0 - x_1, 0])$

For $B = 1$, since the Markov chains are identical, the average age is the same for servers able and unable to harvest energy while a packet is in service. For $B = 2$ the average age can be written as $\Delta_{\mathsf{B}} = X/Y$ where

$$X \triangleq \rho^3(2\beta^4+4\beta^3+3\beta^2+3\beta+1)+\rho^2(2\beta^5+6\beta^4+6\beta^3+3\beta^2+\beta)$$
$$+\rho(2\beta^3+\beta^2)(\beta+1)^2+\beta^3(\beta+1)^2,$$
$$Y \triangleq \mu(\beta+1)[\rho^3\beta(\beta^2+\beta+1)+\rho^2\beta^2(\beta+1)^2+\rho(\beta^4+\beta^3)].$$

Note that the symmetry observed in (4) is no longer present here. For general B, the average age can be written as

$$\Delta_{\mathsf{B}} = \frac{\rho^{B+1}f_0(\beta)+\rho^B f_1(\beta)+\ldots+\rho f_B(\beta)+f_{B+1}(\beta)}{\mu[\rho^{B+1}g_0(\beta)+\rho^B g_1(\beta)+\ldots+\rho g_B(\beta)]}, \quad (9)$$

where $f_i(\beta)$ and $g_j(\beta)$ are polynomial functions of β with degree of at most $2B+1$ for $i \in \{0,1,\ldots,B+1\}$ and $j \in \{0,1,\ldots,B\}$.

The form of (9) allows us to derive an asymptotic result for the case when the status update arrival rate is large. For fixed η, μ, and B, when $\lambda \to \infty$ or, equivalently, $\rho \to \infty$, from (9) it can be shown that

$$\lim_{\rho\to\infty} \Delta_{\mathsf{B}} = \begin{cases} \dfrac{2\beta^2+2\beta+1}{\mu(\beta^2+\beta)} & B = 1 \\ \dfrac{f_0(\beta)}{\mu g_0(\beta)} & B \ge 2 \end{cases}, \quad (10)$$

where

$$f_0(\beta) = 2(\beta+1)\sum_{k=0}^{B+1}\beta^k - (\beta^2+\beta+1),$$
$$g_0(\beta) = (\beta+1)\sum_{k=1}^{B+1}\beta^k.$$

For fixed λ, μ, and B, when the energy arrival rate becomes large, i.e., $\eta \to \infty$ or, equivalently, $\beta \to \infty$, the average age of this model is identical to the case when the server is unable to harvest energy while a packet is in service. Intuitively, this follows from the fact that the battery is always either full or one unit less than full when $\beta \to \infty$. In the steady state, the system is always traversing the states at the rightmost end of the Markov chains, i.e., states $2B-3$, $2B-1$, and $2B$, and there is no advantage in being able to harvest energy during service. Similarly, for fixed λ, η, and B, when the service rate becomes large, the average age of this model is identical to the case when the server is unable to harvest energy while a packet is in service. Intuitively, when $\mu \to \infty$, as soon as a packet is presented to the server, it is instantaneously delivered. Hence, the probability of an energy unit arriving during service becomes small and the Markov chains of the two cases become identical, resulting in the same asymptotic average age.

IV. NUMERICAL RESULTS

This section provides numerical examples to quantify the average age as a function of the system parameters ρ, β, μ and B. Figures 5 and 6 represent the contour plots of the average ages Δ_{A} and Δ_{B}, respectively as a function of the system parameters $0.1 \le \beta \le 10$, $0.1 \le \rho \le 10$, $\mu = 1$ and $B = \{1, 5, 10, 20\}$. From Fig. 5, first, the results confirm the symmetric behavior of Δ_{A} with respect to ρ and β as discussed previously. Second, observe that $\lim_{\substack{\rho\to\infty \\ \beta\to\infty}} \Delta_{\mathsf{A}} = 2/\mu = 2$, which agrees with (5)–(6). Note that the average age in the figures is equivalently a function of η and λ since $\eta = \beta$ and $\lambda = \rho$ for $\mu = 1$.

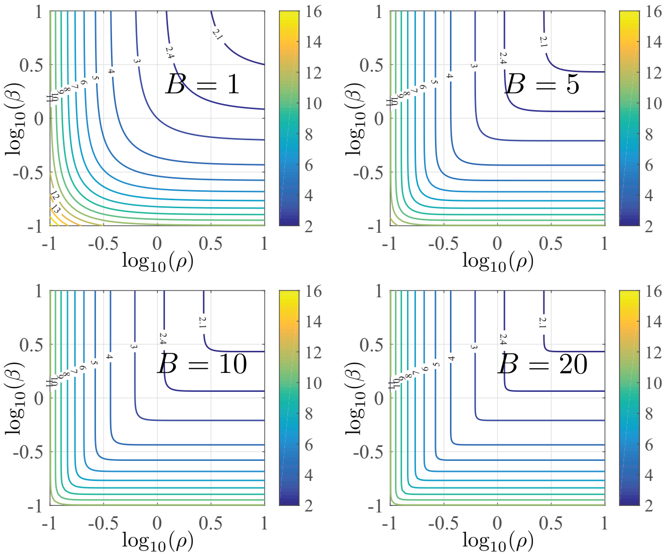

Fig. 5. Contours of the average age of case A where the server cannot harvest energy during service for $0.1 \leq \beta \leq 10$, $0.1 \leq \rho \leq 10$, $\mu = 1$, and $B = \{1, 5, 10, 20\}$.

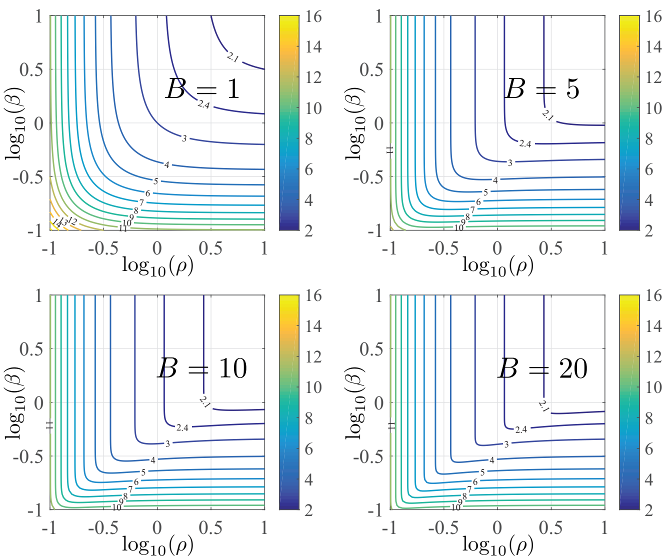

Fig. 6. Contours of the average age of case B where the server can harvest energy during service for $0.1 \leq \beta \leq 10$, $0.1 \leq \rho \leq 10$, $\mu = 1$, and $B = \{1, 5, 10, 20\}$.

Figure 7 represents the ratio of the average age in case B and case A for $B = \{2, 5, 10, 20\}$ and shows the performance improvement in terms of average AoI reduction when the server is able to harvest energy while servicing packets. The results show that as β increases, both cases have the same average age, but when β decreases case B leads to a considerably better average age. When the server can harvest energy during service, as long as the battery is not full, no energy unit is wasted. Consequently we expect case B to have a better average age performance than case A. Since $\Delta_{\mathsf{B}}/\Delta_{\mathsf{A}} \leq 1$ regardless of any of the system parameters, case A provides an upper bound on the average age of case B.

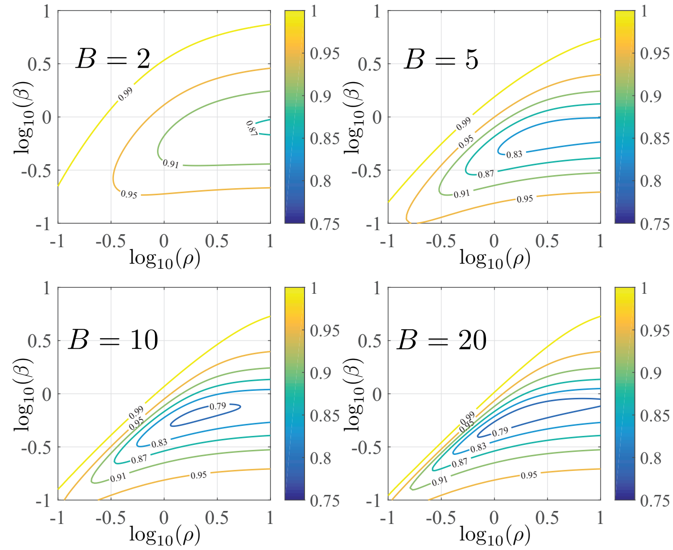

Fig. 7. Contours of the ratio of the average age of cases B and A, i.e., $\Delta_{\mathsf{B}}/\Delta_{\mathsf{A}}$, for $0.1 \leq \beta \leq 10$, $0.1 \leq \rho \leq 10$, $\mu = 1$, and $B = \{2, 5, 10, 20\}$.

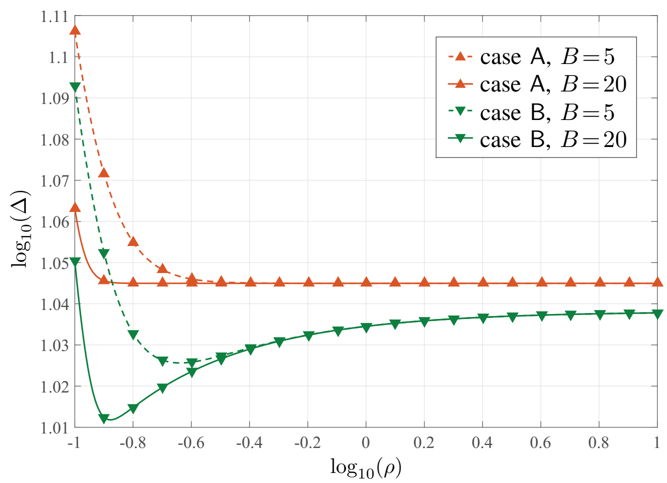

Fig. 8. The average age of cases A and B for $0.1 \leq \rho \leq 10$, $\beta = 0.1$, $\mu = 1$, and $B = \{5, 20\}$.

Figure 8 represents the average age of cases A and B for $\mu = 1$, $\beta = 0.1$, and $B = \{5, 20\}$. The results show that for case A, Δ_{A} is monotonically decreasing with ρ and independent of B. But, for case B, as ρ and B increase, there exists an optimal choice of ρ that minimizes Δ_{B}. This result is similar to a result in [18] where an optimal server utilization rate was shown to minimize the average age. Also, as B increases the average age of the same case decreases monotonically. This is because in general more energy units are stored in the battery and the number of packets that are dropped because of an empty battery is reduced, resulting in more status updates being delivered to the destination, which reduces the average age. Observe that when ρ is comparable to β, $\beta < \rho \ll 1$, the servers ability to harvest energy while a packet is in service for case B leads to a considerable advantage over case A.

V. Conclusion

This paper studied the AoI problem in a single-source status update system with an energy harvesting server with finite battery capacity. Two cases without and with energy harvesting during service were considered and expressions for the average age were derived as a function of the system parameters. Asymptotic average age expressions were also derived for several cases. Numerical results were provided to quantify the average age in terms of the system parameters and numerically demonstrate the performance advantage of servers with simultaneous service and energy harvesting.

Future directions of this work include evaluating the AoI under packet management policies and storing the arriving information packets while the sever is busy. Extending this scenario to a multi-source system where there exist explicit contention among the sources to access the channel resources is another interesting future work.

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