EQUALIZATION WITH PART-TIME HELP

A.G. Klein and P. Duhamel

Laboratoire des Signaux et Systèmes/Supélec/CNRS
3 rue Joliot-Curie
F-91190 Gif-sur-Yvette, France

ABSTRACT

We consider relay-assisted equalization where a half-duplex relay forwards a signal to a receiver in attempt to aid in the task of equalization. All channels are assumed to be frequency selective, and therefore they contribute intersymbol interference (ISI). We first consider the case of a naïve amplify-and-forward protocol where the relay forwards a scaled version of its received (and ISI-corrupted) signal. We then consider an equalize-and-forward protocol where the equalizer attempts to perform linear equalization before forwarding its signal. We show that a relay can indeed provide considerable benefit in the task of equalization even when only providing "part-time help", and we demonstrate the performance of the two schemes with simulations.

***Index Terms*—** half-duplex, relay, amplify-and-forward, equalize-and-forward, intersymbol interference

1. INTRODUCTION

Relay-assisted communication has garnered much interest in recent times, particularly for its use in so-called cooperative diversity [1]. However, relatively little research exists on the use of relays in intersymbol interference (ISI) channels. While the bulk of recent interest in the use of relays has been for their potential for increased diversity in flat-fading Rayleigh channels, here the focus is on the ability of a relay to assist a finite-length symbol-rate linear equalizer in combatting ISI in static frequency-selective channels. It is known that a finite-length symbol-rate linear equalizer cannot perfectly combat ISI in the single-antenna point-to-point scenario [2], and so we investigate whether perfect ISI removal is possible with the assistance of a half-duplex relay. We seek to answer the following questions:

- Does a "dumb" half-duplex amplify-and-forward (AF) relay provide any benefit in the presence of ISI?
- Under what conditions can a linear equalizer at the receiver remove the ISI?
- Is there a performance improvement if the half-duplex relay attempts to remove ISI before forwarding (i.e. equalize-and-forward)?

To the best of our knowledge, these questions have not yet been explored. The possibility of using relays in transmission through ISI and frequency-selective channels has been hinted at in various works in the cooperative diversity literature [3][4], but these previous works have focussed on the problem of dealing with asynchronicity.

We begin by presenting the system model, and then describe the operation of the amplify-and-forward relay in an ISI channel. After addressing the conditions necessary for ISI cancellation, we move on to an equalize-and-forward protocol where the relay performs equalization before forwarding its signal. We demonstrate the mean-square error (MSE) performance of the two protocols via simulations, followed by concluding remarks.

2. SYSTEM MODEL

2.1. Preliminaries

The system model is shown in Fig. 1. We assume that a source transmits a continuous stream of data to a destination, and that a half-duplex relay assists the source in amplifying-and-forwarding the data on a channel orthogonal to the source-destination link. Since the relay is constrained to be half-duplex, we assume that the relay listens for N symbol periods, and then transmits for N symbol periods. As most of the existing work that addresses relaying strategies makes the assumption of a memoryless channel, the choice of frame length N has had little if any importance in frequency non-selective channels, so most previous works simply assume $N = 1$. In that case, the half-duplex relay forwards symbols that are sent from the transmitter during the even time periods, and the relay can provide no help for symbols sent during the odd time periods. The situation is quite a bit different in an ISI channel, however, and the choice of frame length will play a role. If, for example, the channel length is longer than N, the half-duplex relay will forward a signal that contains contributions (in the form of ISI) from *all* transmitted symbols, in spite of the fact that it is only providing part-time help. This suggests that in some cases ISI may enable a relay to assist in forwarding all of the symbols, even if the relay is constrained to be half-duplex.

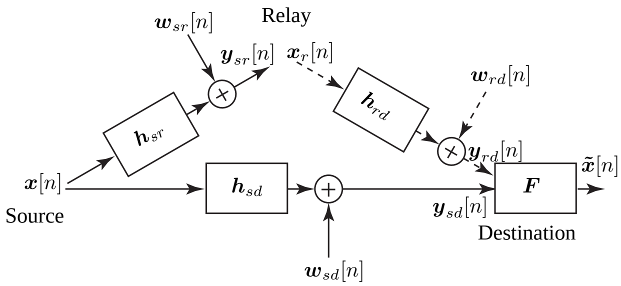

Fig. 1. System Model

We consider a simple example to illustrate, focussing on the source-relay link. Take $N = 1$ and let the channel between the source and relay have impulse response $[1, \frac{1}{2}]$, so that its length exceeds N. If the source transmits the BPSK symbols $[+1, -1, +1, +1, -1, -1]$, for example, the sequence received at the relay will be the convolution which is given by $[1, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, -\frac{1}{2}, -\frac{3}{2}, -\frac{1}{2}]$ plus noise. However, since the half-duplex relay can only receive half of the time, the received sequence at the relay is decimated and therefore becomes $\frac{1}{2}[-1, +3, -3]$. Thus, for this particular example, the combined effect of this ISI channel and decimation at the relay can be seen as re-modulation of two BPSK symbols into a single scaled 4-PAM symbol, via the pairwise mapping $(+1, +1) \to +3$, $(-1, +1) \to +1$, $(+1, -1) \to -1$, $(-1, -1) \to -3$. Under the amplify-and-forward protocol, then, the destination receives the original rate $1/T$ BPSK symbols directly from the source, and a signal of rate $1/2T$ 4-PAM symbols from the relay. Both of these signals contain all of the transmitted symbols, in spite of the fact that the relay is constrained to be half duplex. While this is a contrived example unlikely to arise in practice, it demonstrates the *possibility* that a half-duplex relay forwarding ISI-corrupted data can assist the destination during all symbol periods.

While we do not treat the issue of frame-synchronization directly, it is closely related to the issue of ISI as evidenced by the number of previous works [3][4] which have proposed equalizer-like structures for handling the asynchronicity of the relay. We note, however, that our model is general enough to include such asynchronicity by adding zeros (i.e. delays) to the front of the source-relay channel response. We make no assumptions that any form of frame-synchronization has been performed, nor that the relay has knowledge of its time offset relative to the source. Thus, in the $N = 1$ case, for example, our naïve relay may assist in forwarding symbols sent at even time slots (in the case of a trivial unity channel), at odd time slots (for a channel that is a simple bulk delay), or a mix of symbols where being “frame-synchronous” is not well defined (in ISI channels). Finally, we assume that receivers can acquire the channel impulse response (for example, through an initial training period), but we do not assume there is any feedback that would enable the transmitters to have any channel information.

2.2. Amplify-and-Forward

We now describe the details of the system model shown in Fig. 1. We assume that the relay is a naïve amplify-and-forward device which satisfies an average unit-power constraint. The source itself is not constrained to any frame structure *per se*, as it simply sends a constant stream of data. The relay, on the other hand, effectively introduces a frame structure of length $2N$ symbols since it listens for N symbols, and then re-transmits the scaled received signal during the next N time slots. Thus, we will employ a model based on this frame structure, where the time index n is at the frame rate.

The destination employs a frame-rate linear filter $\boldsymbol{F} \in \mathbb{C}^{N_f \times 2N}$ which performs equalization and combining of the two signals received from the source and relay. The N_f rows of this matrix shall be partitioned into one group of N_{f1} rows that equalize the signal from the source, and one group of N_{f2} rows that equalize the signal from the relay, so that $N_f = N_{f1} + N_{f2}$. Thus, the length $2N$ frame output of this filter is

$$\tilde{\boldsymbol{x}}[n] \quad = \quad \boldsymbol{F}^\top \begin{bmatrix} \boldsymbol{y}_{sd}[n] \\ \boldsymbol{y}_{rd}[n] \end{bmatrix} \in \mathbb{C}^{2N} \qquad (1)$$

where $\boldsymbol{y}_{sd}[n] \in \mathbb{C}^{N_{f1}}$ is the signal received from the source, and $\boldsymbol{y}_{rd}[n] \in \mathbb{C}^{N_{f2}}$ is the signal received from the relay.

We assume that the source sends i.i.d. complex symbols with unit average power, and that all channels are causal linear time-invariant FIR with complex circularly symmetric additive white Gaussian noise (AWGN). The source-destination, source-relay, and relay-destination channel impulse responses are denoted by $\boldsymbol{h}_{sd}, \boldsymbol{h}_{sr}, \boldsymbol{h}_{rd}$, respectively (where for example $\boldsymbol{h}_{sd} = [h_{sd}[0] \ \ldots \ h_{sd}[N_{sd} - 1]]^\top$), and they have corresponding lengths N_{sd}, N_{sr}, N_{rd} and AWGN noise powers $\sigma_{sd}^2, \sigma_{sr}^2, \sigma_{rd}^2$. We can then write the corresponding received signals as

$$\boldsymbol{y}_{sd}[n] \quad = \quad \boldsymbol{\mathcal{H}}_{sd}\boldsymbol{x}[n] + \boldsymbol{w}_{sd}[n] \qquad (2)$$

$$\boldsymbol{y}_{sr}[n] \quad = \quad \boldsymbol{\mathcal{H}}_{sr}\boldsymbol{x}[n] + \boldsymbol{w}_{sr}[n] \qquad (3)$$

$$\boldsymbol{y}_{rd}[n] \quad = \quad \boldsymbol{\mathcal{H}}_{rd}\boldsymbol{x}_r[n] + \boldsymbol{w}_{rd}[n] \qquad (4)$$

where $\boldsymbol{x}[n] = [x[2Nn], x[2Nn-1], \ldots] \in \mathbb{C}^{N_{f1}+N_{sd}-1}$ contains the transmitted symbols, $\boldsymbol{\mathcal{H}}_{sd} \in \mathbb{C}^{N_{f1} \times N_{f1}+N_{sd}-1}$, $\boldsymbol{\mathcal{H}}_{sr} \in \mathbb{C}^{N_{f2}+N_{rd}-1 \times N_{f2}+N_{rd}+N_{sr}-2}$, and $\boldsymbol{\mathcal{H}}_{rd} \in \mathbb{C}^{N_{f2} \times N_{f2}+N_{rd}-1}$ are the Tœplitz channel convolution matrices defined, for example, as $[\boldsymbol{\mathcal{H}}_{sd}]_{i,j} = h_{sd}[j - i]$, and $\boldsymbol{w}_{\{sd,sr,rd\}}[n]$ is the AWGN. The channel impulse responses include the pulse shaping, possible frame asynchronicity, and carrier phase offsets. So that our matrices contain whole frames, we implicitly require $N_{f2} + N_{rd} - 1$ to be a multiple of $2N$, and that $N_{f1} + N_{sd} + 1 = N_{f2} + N_{rd} + N_{sr}$, both of which can trivially be satisfied by appending zeros to the appropriate channel impulse responses.

The signal $\boldsymbol{x}_r[n] \in \mathbb{C}^{N_{f2}+N_{rd}-1}$ which appears in (4) is the signal emitted by the amplify-and-forward relay. This can be expressed as $\boldsymbol{x}_r[n] = \beta\boldsymbol{\Gamma}\boldsymbol{y}_{sr}[n]$ where the scale factor

$\beta = 1/\sqrt{||\boldsymbol{h}_{sr}||_2^2 + \sigma_{sr}^2}$ is chosen to satisfy the average unit-power constraint, and the square matrix $\boldsymbol{\Gamma}$ is given by

$$\boldsymbol{\Gamma} \quad = \quad \boldsymbol{I}_{(N_{f2}+N_{rd}-1)/2N} \otimes \begin{bmatrix} \boldsymbol{0}_{N\times N} & \boldsymbol{I}_N \\ \boldsymbol{0}_{N\times N} & \boldsymbol{0}_{N\times N} \end{bmatrix} \qquad (5)$$

with the role of $\boldsymbol{\Gamma}$ being the removal of samples from $\boldsymbol{y}_{sr}[n]$ (to impose the half-duplex constraint during reception to the relay), delaying by N symbols, and reinsertion of zeros (to impose the half-duplex constraint during transmission from the relay). Thus, the equalizer output can be written

$$\tilde{\boldsymbol{x}}[n] \quad = \quad \boldsymbol{F}^\top \left(\boldsymbol{\mathcal{H}}_{eff}\boldsymbol{x}[n] + \boldsymbol{w}_{eff}[n] \right) \qquad (6)$$

where

$$\boldsymbol{\mathcal{H}}_{eff} \quad = \quad \begin{bmatrix} \boldsymbol{\mathcal{H}}_{sd} \\ \beta\boldsymbol{\mathcal{H}}_{rd}\boldsymbol{\Gamma}\boldsymbol{\mathcal{H}}_{sr} \end{bmatrix} \qquad (7)$$

$$\boldsymbol{w}_{eff}[n] \quad = \quad \begin{bmatrix} \boldsymbol{w}_{sd}[n] \\ \boldsymbol{w}_{rd}[n] + \beta\boldsymbol{\mathcal{H}}_{rd}\boldsymbol{\Gamma}\boldsymbol{w}_{sr}[n] \end{bmatrix}. \qquad (8)$$

For a chosen frame delay Δ through the channel-equalizer chain, we form the mean-squared error (MSE) as

$$MSE = E\left[||\tilde{\boldsymbol{x}}[n] - \boldsymbol{E}_\Delta^\top \boldsymbol{x}[n]||_2^2 \right] \qquad (9)$$

where

$$\boldsymbol{E}_\Delta = \begin{bmatrix} \boldsymbol{0}_{2N\times 2N\Delta} & \boldsymbol{I}_{2N} & \boldsymbol{0}_{2N\times N_{f1}+N_{sd}-1-2N-2N\Delta} \end{bmatrix}^\top.$$

The orthogonality principle gives the MMSE filter as

$$\boldsymbol{F}_{mmse}^* = (\boldsymbol{\mathcal{H}}_{eff}\boldsymbol{\mathcal{H}}_{eff}^H + \boldsymbol{R}_{ww})^{-1}\boldsymbol{\mathcal{H}}_{eff}\boldsymbol{E}_\Delta \qquad (10)$$

where $\boldsymbol{R}_{ww} = E\left[\boldsymbol{w}_{eff}[n]\boldsymbol{w}_{eff}^H[n] \right]$. We note that the zero-forcing (ZF) equalizer can be calculated using (10) but taking $\boldsymbol{R}_{ww} = \boldsymbol{0}$.

We now step back and make several observations. It is well-known that a finite-length equalizer cannot perfectly remove ISI in a single antenna non-oversampled point-to-point system [2], primarily because the channel matrix is not tall. Indeed, $\boldsymbol{\mathcal{H}}_{sd}$ is a wide matrix, and so channel inversion is not possible without the relay. However, the addition of a naïve amplify-and-forward relay has the effect of making the effective channel matrix (7) tall, which implies the possibility that the finite-length equalizer can *completely* remove all ISI (e.g. with a ZF equalizer). Specifically, the effective channel matrix needs to be full rank, which is the case when there are no common roots among the two subchannels [2], i.e. between the source-destination channel $\boldsymbol{\mathcal{H}}_{sd}$, and the combined source-relay-destination channel $\boldsymbol{\mathcal{H}}_{rd}\boldsymbol{\Gamma}\boldsymbol{\mathcal{H}}_{sr}$. In addition, a tall channel matrix implies that blind subspace-based channel identification techniques can be used [5] when the rank condition is satisfied. Furthermore, since the MSE in (9) is quadratic in $\boldsymbol{F}$, we can use stochastic gradient descent algorithms such as Least Mean Squares (LMS) to adaptively determine the equalizer coefficients of the MMSE equalizer.

Thus, the addition of a naïve amplify-and-forward relay operating in a half-duplex mode can permit perfect ISI removal with a linear receiver in cases where perfect ISI removal would not otherwise have been possible. In addition, determination of the MMSE filter taps can be done very easily with LMS, or with blind subspace-based channel identification methods. Furthermore, we reiterate that the relay itself does not need to perform any frame-synchronization – though the receiver needs to estimate the effective channel which may include propagation delays. Finally, we point out that the MMSE equalizer given by (10) coincides with the maximal ratio combiner (MRC) when all the channels are non-frequency selective (i.e. a single tap).

2.3. Equalize-and-Forward

The previous section demonstrated that the naïve amplify-and-forward protocol can bring about significant benefits in an ISI channel because a linear equalizer can in some cases perfectly cancel the ISI. We now consider an equalize-and-forward protocol where the relay performs linear MMSE equalization with the filter $\boldsymbol{G} \in \mathbb{C}^{N_g\times N}$ before forwarding its signal. While we could choose $\boldsymbol{G}$ to minimize the global MSE given in (9), this would imply that the relay has access to the relay-destination channel coefficients which would require some feedback mechanism. A global MSE minimization would likely lead to better system performance, but we do not assume the existence of a feedback channel in our system model, and so we choose $\boldsymbol{G}$ to minimize the MSE between the source and the relay output. Thus, the goal of the equalizer at the relay will be to mitigate ISI introduced on the source-relay channel. It is not clear *a priori* if such equalization at the relay is necessarily useful; as suggested by the example in section 2.1, residual ISI may at times be beneficial since it can permit a half-duplex relay to assist the destination in decoding *all* symbols, even though it only participates half of the time. Furthermore, since the half-duplex relay only receives half of the time, it is not clear if an equalizer operating on a decimated signal can really succeed in equalizing its received signal. In fact, the decimation at the relay results in an effective source-relay channel $\boldsymbol{\Gamma}\boldsymbol{\mathcal{H}}_{sr}$ that is very wide, which means that perfect equalization at the relay is not possible. Nevertheless, we adopt this protocol *faute de mieux* for comparison with the amplify-and-forward protocol.

As the system model is quite similar, we do not present the equations for the system model due to space constraints. The only change is that the relay performs equalization of its decimated input with the filter $\boldsymbol{G}$ before forwarding. We now derive the MMSE filter taps for $\boldsymbol{G}$ which minimize the MMSE at the output of the relay. Letting Δ_r be the designer-chosen delay through the source-relay channel and relay equalizer, the MSE becomes

$$MSE_r = E\left[||\boldsymbol{G}^\top(\boldsymbol{I} \otimes [\boldsymbol{0}_{N\times N}\boldsymbol{I}_N])\boldsymbol{y}_{sr}[n] - E_{\Delta'}\boldsymbol{x}[n]||_2^2 \right]$$

where $E_{\Delta'}$ is defined as in (10). Again, the orthogonality principle gives

$$\boldsymbol{G}^*_{mmse} = \left(\boldsymbol{\mathcal{H}}_{eff,r}\boldsymbol{\mathcal{H}}^H_{eff,r} + \sigma^2_{sr}\boldsymbol{I}_{N_g}\right)^{-1}\boldsymbol{\mathcal{H}}_{eff,r}\boldsymbol{E}_{\Delta_r}$$

where

$$\boldsymbol{\mathcal{H}}_{eff,r} \quad = \quad (\boldsymbol{I} \otimes [\boldsymbol{0}_{N\times N}\boldsymbol{I}_N])\boldsymbol{\mathcal{H}}_{sr}$$

is the effective source-relay channel after decimation. We note that under the equalize-and-forward protocol, the end-to-end effective channel matrix given by (7) as well as the equation for the MMSE equalizer $\boldsymbol{F}$ in (10) need to be modified to include the equalizer at the relay, which can be accomplished by insertion of $\boldsymbol{G}^\top$ in the appropriate places. Next, we investigate the performance of each of these protocols.

3. SIMULATIONS

We now simulate the MSE performance of these schemes with the following parameters: the symbols are chosen to be QPSK, the block length is $2N = 4$, the noise power is assumed to be equal on all channels (representing a situation where the source, relay, and destination are equidistant), the channel lengths are $N_{sd} = N_{sr} = N_{rd} = 3$, the equalizer lengths are $N_{f1} = N_{f2} = N_g = 6$, and the chosen combined channel/equalizer delays are set at $\Delta = \Delta_r = 0$. The 3 taps on each of the 3 channels are i.i.d. circular symmetric Gaussian variables (i.e. Rayleigh distributed), where the 3 taps have variance (i.e. power decay profile) given by [0.59, 0.29, 0.12]. Thus, while the noise power is identical on all channels, some may be in deep fade depending on the fading coefficients. We averaged over 10,000 channels, and plotted the MSE performance of each of the protocols in Fig. 2. In addition, we have included the performance of the classical MMSE equalizer with no relay – and for fairness we allot all $N_{f1} + N_{f2}$ equalizer taps to equalization of $\boldsymbol{y}_{sd}[n]$ in this case.

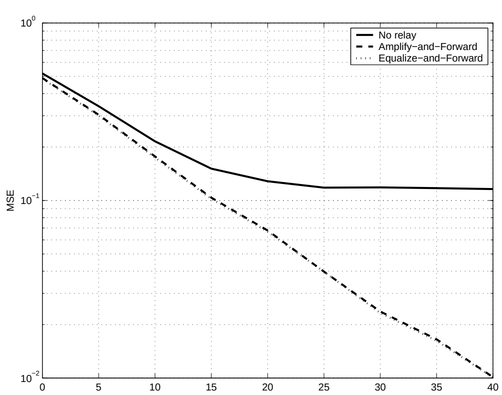

Fig. 2. MSE Performance Averaged

Note that it is nearly impossible to discern any difference between the amplify-and-forward and the equalize-and-forward protocols, as the two curves lie on top of one another. We have performed countless other simulations with varied system parameters, and this seems to be uniformly true. Thus, it is questionable whether the addition of an equalizer at the relay is really worth the added complexity. If the relay could operate in full-duplex mode, or if the destination could feedback information about the relay-destination channel, than the situation would certainly be different.

In the absence of a relay, we observe that the performance of the equalizer reaches a floor. Because the non-relay-assisted equalizer is unable the mitigate all of the ISI, its performance is considerably worse at high SNR. The MSE of the relay-assisted equalization, on the other hand, decays with SNR.

4. CONCLUSION

We have examined the use of a half-duplex relay in assisting with the task of linear equalization. We showed that, with some assumptions about channel rank, a naïve amplify-and-forward relay can enable perfect ISI removal in situations where it would not have otherwise been possible. We also considered an equalize-and-forward protocol, and through simulations we showed that this protocol seems to provide little if any benefit over the simpler amplify-and-forward protocol. Future work will investigate equalization in the case where the relay channel is not orthogonal to the source-destination channel, as well as receiver structures for efficiently exploiting the diversity offered by fading ISI channels in relay-assisted scenarios.

5. REFERENCES

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[2] L. Tong, G. Xu, and T. Kailaith, “Fast blind equalization via antenna arrays,” in *Proc. IEEE Intl. Conf. on Acoustics, Speech, and Signal Processing (ICASSP’93)*, Apr. 1993, pp. 272–275.

[3] S. Wei, D. Goeckel, and M. Valenti, “Asynchronous cooperative diversity,” IEEE Trans. Wireless Commun., to appear.

[4] X. Li, F. Ng, J. Hwu, and M. Chen, “Channel equalization for STBC-encoded cooperative transmissions with asynchronous transmitters,” in *Proc. Asilomar Conf. on Signals, Systems, and Computers*, Oct. 2005.

[5] E. Moulines, P. Duhamel, J. Cardoso, and S. Mayrargue, “Subspace methods for the blind identification of multichannel FIR filters,” *IEEE Trans. Signal Processing*, vol. 43, pp. 516–525, Feb. 1995.