Decision Feedback Sparsening Filter Design for Belief Propagation Detectors

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***Abstract*—A large body of research exists around the idea of channel shortening, where a prefilter is designed to reduce the effective channel impulse response to some smaller number of contiguous taps. This idea was originally conceived to reduce the complexity of Viterbi-based maximum likelihood equalizers. Here, we consider a generalization of channel shortening which we term "channel sparsening." In this case, a decision feedback filter is designed to reduce the effective channel to a small number of nonzero taps which do not need to be contiguous. When used in combination with belief propagation-based maximum *a posteriori* detectors, an analogous complexity reduction can be realized. We address the design aspects of decision feedback sparsening filters, devote attention to the interaction of the sparsening filter and detector, and demonstrate the performance gains through simulation.**

Key words: belief propagation, turbo equalization, channel sparsening, channel shortening.

I. Introduction

As the requirement for higher data rates increases, the design of communication systems that are able to mitigate the effects of frequency selective channels is still a challenge for communication engineers. In high data rate systems, the transmitted symbols are subjected to a phenomenon known as intersymbol interference (ISI), and a wide range of strategies are available for use by communication system designers. For optimal performance, a maximum *a posteriori* (MAP) or maximum-likelihood (ML) sequence estimator may be implemented using the Bahl-Cocke Jelinek-Raviv (BCJR) or Viterbi algorithm, respectively. These optimal approaches, however, are exponentially complex in the number of channel coefficients, and therefore not practical in most cases.

Sparse impulse responses are characterized as having only a small fraction of nonzero coefficients. This behavior can arise, for example, in underwater acoustic communication channels or in terrestrial communication channels over hilly terrain. Compensation of sparse ISI channels is considerably challenging since these channels can often have very long delay spreads, and optimal approaches like BCJR and Viterbi are therefore infeasible. Recently, a MAP detector employing belief propagation (BP) was proposed [1] for ISI compensation in sparse channels. The proposed scheme is attractive because it permits near-optimal performance with complexity that depends only on the number of nonzero coefficients. The complexity of this algorithm is *exponential* in the number of nonzero channel coefficients, however, so it may still be prohibitively complex for the majority of applications. A hybrid version of this detector was proposed [2] which uses a linear prefilter in the receiver just before the BP-based MAP detector. By designing the prefilter so that the combined response of the sparse channel and prefilter has a reduced, limited number of nonzero coefficients, the use of the BP-based detector becomes feasible in a wider range of applications.

In this work, we focus on the design of decision feedback *sparsening* filters for use with soft-input soft-output MAP detectors of the form considered in [1], [2]. In particular, we extend the hybrid structure proposed by [2] so that the linear sparsening filter which performs partial equalization is replaced by a non-linear decision feedback sparsening filter. While [1], [2] primarily focused on the case where the original channel is sparse, we note that even non-sparse channels can be sparsened with this approach. Consequently, our work can be applied in general situations, even where the original channel is not sparse. We address the issue of sparsening filter design, we consider the interaction of the sparsening filter and BP detector, and we develop a practically-implementable sparsening filter design method.

We note that channel sparsening filters can be seen as a generalization of so-called *channel shortening filters* proposed in [3]–[8]. Given an finite impulse response (FIR) channel $\boldsymbol{h}$ of length L_h, the channel shortening problem roughly amounts to designing a filter so that the energy in the combined response of the channel and filter is concentrated in $\mu \leq L_h$ *contiguous* taps. Channel sparsening is nearly the same, though the μ taps which contain the majority of the energy are not constrained to be contiguous. Furthermore, while much of the recent interest in channel shortening has been for application to multicarrier systems, the original idea of channel shortening [3], [4] was proposed for a reduced-complexity hybrid prefilter/ML detector which bears some resemblance to the one considered here. More recent works such as [9] have considered channel

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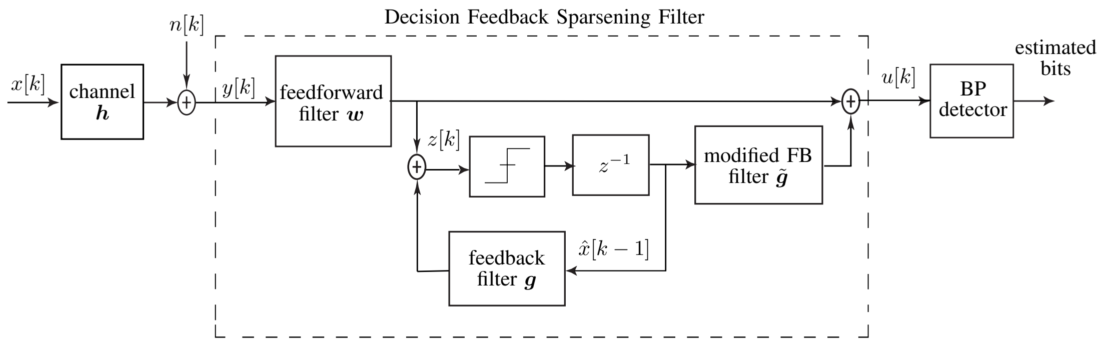

Fig. 1. System Model

shortening in conjunction with iterative MAP detectors. Again, however, these works impose a constraint that the taps in the combined channel/filter response must be contiguous. Two very recent works have considered the design of sparsening filters where the taps are non-contiguous. In [10], the design of linear sparsening filters was considered, and a design approach was developed that attempted to optimize a proxy for the bit error rate. In [11], the authors considered the use of matching pursuit to find a sparse, non-contiguous target impulse response, and it was shown to yield a lower mean squared error (MSE) compared to the conventional contiguous approach.

II. System Model

As mentioned, previous works [2], [10] have investigated use of *linear* prefilters designed to sparsen an ISI channel, thereby decreasing the complexity of the detection process so that BP-based detectors can be implemented. It is known, however, that linear filtering enhances noise, particularly when the frequency response of the desired sparsened channel differs greatly from the true channel [10]. To improve upon these works which employ linear filters to perform channel sparsening, we leverage the rich body of knowledge in decision feedback equalizers (DFEs) which are known to outperform linear equalizers in a wide range of practical scenarios. By embedding a DFE in the channel sparsening filter, we arrive at a novel structure that sparsens the channel through feedback of symbol decisions which subtract off residual ISI.

We consider the system model shown in Fig. 1. A sequence of symbols $x[k]$ drawn from an M-ary alphabet is transmitted through an ISI channel denoted $\boldsymbol{h}$ – which may or may not be sparse – and additive white Gaussian noise $n[k]$ with variance σ_n^2 is added. At the receiver, we employ a detector which consists of the cascade of the proposed decision-feedback sparsening filter (DFSF) followed by a BP-based detector [1]. The DFSF consists of a feedforward filter $\boldsymbol{w}$, a memoryless decision device (or slicer), a feedback (FB) filter $\boldsymbol{g}$, as well as a modified FB filter $\tilde{\boldsymbol{g}}$ whose output is added to the output of the feedforward filter to serve as input to the BP detector.

As mentioned, the BP detector is exponentially complex in the number of nonzero channel taps. Consequently, the purpose of the DFSF is to reduce the number of nonzero coefficients in the effective channel to some specified quantity μ so that the use of the BP detector becomes practically feasible. The choice of μ poses as a complexity constraint on the number of ISI taps compensated by the BP detector. Smaller values of μ put more of the burden in ISI compensation on the DFSF, whereas larger values put more burden on the BP detector. We note that the DFSF includes a regular DFE comprised of the feedback loop with filters $\boldsymbol{w}$ and $\boldsymbol{g}$ that ideally suppress all ISI, enabling the slicer to make tentative decisions. These tentative decisions are filtered by $\tilde{\boldsymbol{g}}$ and added to the information corrupted by the ISI channel and the feedforward filter to result in the controlled ISI to be mitigated by the BP detector. While we delay discussion of the design of $\boldsymbol{w}$, $\boldsymbol{g}$, and $\tilde{\boldsymbol{g}}$ until section IV, we note that a structure similar to the DFSF was considered in the classic channel-shortening literature [4], though in that case the locations of the residual ISI taps were constrained to be contiguous. By setting the first $\mu - 1$ taps of $\tilde{\boldsymbol{g}}$ to zero, the channel shortener in [4] leaves μ contiguous uncompensated taps in the effective channel which are compensated by a Viterbi detector.

We assume that the channel, the feedforward filter, and the feedback filters are modeled as FIR filters of lengths L_h, L_w, L_g, and $L_{\tilde{g}} = L_g$, respectively. Thus, the received data is given by

$$y[k] = \sum_{l=0}^{L_h-1} h[l]x[k-l] + n[k]. \tag{1}$$

After filtering the signal by the feedforward filter and adding the contribution of the feedback filter, the signal input to the slicer is given by

$$z[k] = \sum_{l=0}^{L_w-1} w[l]y[k-l] + \sum_{l=0}^{L_g-1} g[l]\hat{x}[k-l-1]$$

where $\hat{x}[k]$ are the unreliable tentative decisions output from the slicer. Similarly, the output of the DFSF can be written as:

$$u[k] = \sum_{l=0}^{L_w-1} w[l]y[k-l] + \sum_{l=0}^{L_g-1} \tilde{g}[l]\hat{x}[k-l-1] \tag{2}$$

which gets passed to the soft-input soft-output BP detector that outputs likelihood values that can be used to make decisions as to what was transmitted.

Under the optimistic assumption that the slicer makes correct symbol decisions, the output of the slicer is equal to a delayed version of the transmitted symbols, or $\hat{x}[k] = x[k-\Delta]$ where Δ is the symbol delay. In this case, the output of the DFSF can be written as

$$u[k] \quad = \quad \sum_{l=0}^{L_c-1} c[l]x[k-l] + v[k] \tag{3}$$

where $c[k] \triangleq (h[k] \star w[k]) + \tilde{g}[k-\Delta-1]$ is the combined response of the channel and DFSF having length $L_c = L_h + L_w - 1$, and the DFSF output noise, which is colored, is given by $v[k] = n[k] \star w[k]$. We use the $\star$ operator to denote convolution.

In this work, we focus our attention on the design of the sparsening filter. As such, we make the simplifying assumption that the channel $\boldsymbol{h}$ is known perfectly to the receiver. It is rather straightforward, however, to extend our proposed design method to adaptive implementations which can be employed in situations where the channel is unknown and/or slowly time-varying.

III. Belief Propagation Detector

Before discussing the sparsening filter design in detail, we first provide some details about the BP detector. The BP algorithm used in the detector is in the class of message passing algorithms, and is sometimes called the *sum-product algorithm* [1]. By representing the ISI channel as a factor graph, the BP algorithm can be used to implement MAP detection, thereby finding the sequence $\boldsymbol{x}$ which maximizes the joint *a posteriori* probability density function $P(\boldsymbol{x}|\boldsymbol{y})$. The BP algorithm proceeds iteratively, and computes log likelihood ratios of the transmitted bits which become more reliable with each iteration. After a sufficient number of iterations, the log likelihood ratios can be used to make bit decisions.

To compute the likelihood ratios, the BP detector needs to know the effective channel impulse response (i.e. the combined response of the channel and DFSF under the assumption of correct tentative decisions). Given a finite-length filter $\boldsymbol{h}$, the combined response of the channel and DFSF observed by the BP detector may contain more than μ non-zero taps since the DFSF may not be able to perfectly zero all but μ of the effective channel taps. That is, due to the possibility of residual, uncanceled ISI, it may not be possible to perfectly sparsen the channel so that the effective channel consists of only μ nonzero taps. Nevertheless, to keep computational complexity at the level prescribed by the choice of μ, we only use the largest μ taps of the effective sparsened impulse response in the computation of the likelihood ratios used inside the BP detector. As such the residual ISI contribution from the smallest $L_c - \mu$ taps of $\boldsymbol{c}$ in (3) will be treated as noise by the BP detector. A sufficiently large choice of DFSF lengths L_w and $L_g = L_{\tilde{g}}$, however, can ensure arbitrarily small additional ISI.

Since the BP detector is typically implemented in the log domain, the majority of its complexity is due to the many summation operations which must be performed [1]. If the BP algorithm proceeds over N total iterations, the total complexity requires on the order of $N(\mu+1)M^{\mu+1}$ summations, where M is the size of the source alphabet and μ is the number of significant effective channel taps used in the detection. As such, the complexity of the BP is exponential in μ, and so the system designer can specify the total complexity by appropriate choice of μ.

We note that the BP detector performance only truly coincides with the MAP detector when two conditions are met: 1) there are no cycles in the factor graph corresponding to the channel, and 2) the additive noise is white and Gaussian. In general, the first of these conditions is never satisfied. In practice cycles have been shown to be of little concern since they are a low probability event (in the case of potentially detrimental length 4 cycles) [12], or the cycles themselves do not pose a noticeable performance penalty [1]. The second condition on the noise, however, is more serious for this hybrid structure. Since the additive white noise gets colored by the feedforward filter, the noise at the input of the BP detector is no longer white. We will address this issue in the sequel.

We emphasize that the DFSF does not change the operation of the BP detector. As the DFSF changes the effective channel taps, however, and passes the μ largest effective channel taps to the BP detector, the DFSF obviously affects the behavior and performance of the combined filter/detector structure. Since the BP detector itself is unaltered from [1], it can accommodate a system employing channel codes such as LDPC encoding considered in [1], or can readily be extended to the MIMO case with, for example, space-time coding as in [2], [9]. Since our focus is on the design of the DFSF, we consider an uncoded system.

IV. Decision feedback channel sparsening

In the design of the DFSF filters $\boldsymbol{w}$, $\boldsymbol{g}$, and $\tilde{\boldsymbol{g}}$, the goal is for the number of significant (nonzero) taps of the effective channel $\boldsymbol{c}$ to be μ or less, regardless of where they lie in $\boldsymbol{c}$ or whether they are contiguous or not. We note that $\mu \in \{1, 2, \ldots, L_h\}$ is a parameter chosen by the system designer. If $\mu = 1$, then the detector coincides with a traditional DFE since the goal of the DFSF design is to make the effective channel be a single spike. At the other extreme, the choice $\mu = L_h$ corresponds to "pure" BP detection as in [1] since the DFSF need not do any sparsening and can be a simple unity gain filter $\boldsymbol{w}$ and a zeroed $\tilde{\boldsymbol{g}}$. Large choices of μ will result in an exponentially more complicated BP detector, but will also result in better bit error rate (BER) performance.

We now address the design of the filter coefficients $\boldsymbol{w}$, $\boldsymbol{g}$, and $\tilde{\boldsymbol{g}}$. As the DFSF contains an embedded DFE and requires tentative decisions from the memoryless slicer, we choose $\boldsymbol{w}$ and $\boldsymbol{g}$ so that the input to the slicer is (nearly) ISI-free. Toward that end, we adopt the popular minimum mean-squared error (MMSE) criterion to design the feedforward and feedback filters which cancel all of the ISI in a suboptimal manner. To shift some of the burden in ISI compensation from the DFE to the more sophisticated BP-detector, the DFSF has another signal path leading to the BP detector where a filter $\tilde{\boldsymbol{g}}$ is designed so that μ taps of controlled residual ISI are left in the effective channel. As we will discuss, we can derive $\tilde{\boldsymbol{g}}$ from the MMSE choice of $\boldsymbol{g}$ so that the combined response of the channel and DFSF results in an effective channel with μ taps of residual ISI.

First we define

$$\boldsymbol{x}[k] = [x[k],\ x[k-1],\ \ldots,\ x[k-L_w-L_h+2]]^\top$$

and

$$\boldsymbol{\Sigma}_\Delta \triangleq \left[\boldsymbol{0}_{L_g\times(\Delta+1)} \quad \boldsymbol{I}_{L_g} \quad \boldsymbol{0}_{L_g\times(L_w+L_h-\Delta-L_g-2)}\right].$$

Also, under the assumption that the output of the slicer consists of correct decisions delayed by an amount Δ, we have $\hat{x}[k] = x[k-\Delta]$ and can write the vector of symbol decisions with length L_g at time $k-1$ as

$$\hat{\boldsymbol{x}}[k-1] = \boldsymbol{\Sigma}_\Delta \boldsymbol{x}[k].$$

Let $\boldsymbol{H}$ be the $L_w \times L_c$ wide convolution matrix of $\boldsymbol{h}$, i.e. a Tœplitz matrix with first row $[\boldsymbol{h}^\top, \boldsymbol{0}_{1\times L_w-1}]$ and first column $[h[0], \boldsymbol{0}_{1\times L_w-1}]^\top$. As an example, for $L_w = 3$, $L_h = 3$

$$\boldsymbol{H} = \begin{bmatrix} h[0] & h[1] & h[2] & 0 & 0 \\ 0 & h[0] & h[1] & h[2] & 0 \\ 0 & 0 & h[0] & h[1] & h[2] \end{bmatrix}.$$

Then, the slicer input can be rewritten as

$$z[k] = \boldsymbol{w}^\top(\boldsymbol{H}\boldsymbol{x}[k] + \boldsymbol{n}[k]) + \boldsymbol{g}^\top \hat{\boldsymbol{x}}[k-1].$$

And, under the assumption of correct slicer decisions, we can write the DFSF output as

$$u[k] = \boldsymbol{c}^\top \boldsymbol{x}[k] + v[k],$$

as in (3), where $\boldsymbol{c} = \boldsymbol{H}^\top \boldsymbol{w} + \boldsymbol{\Sigma}_\Delta^\top \tilde{\boldsymbol{g}}$ and $v[k] = \boldsymbol{w}^\top \boldsymbol{n}[k]$. To find the MMSE DFE, we minimize the cost function

$$\begin{aligned} J_{mse}(\boldsymbol{w}, \boldsymbol{g}) &= E\left[|z[k] - x[k-\Delta]|^2\right] \\ &= \boldsymbol{w}^\top(\boldsymbol{H}\boldsymbol{H}^\top + \sigma_n^2 \boldsymbol{I}_{L_w})\boldsymbol{w} + 2\boldsymbol{w}^\top \boldsymbol{H}\boldsymbol{\Sigma}_\Delta^\top \boldsymbol{g} \\ &\quad + \boldsymbol{g}^\top \boldsymbol{g} - 2\boldsymbol{w}^\top \boldsymbol{H} \boldsymbol{e}_\Delta + 1 \end{aligned}$$

over $\boldsymbol{w}$ and $\boldsymbol{g}$, where

$$\boldsymbol{e}_\Delta = \left[\underbrace{0,\ \ldots,\ 0}_{\Delta},\ 1,\ \underbrace{0,\ \ldots,\ 0}_{L_w+L_h-\Delta-2}\right]^\top$$

and $x[k-\Delta] = \boldsymbol{e}_\Delta^\top \boldsymbol{x}[k]$. Note that we assume that the source has unit power, and that the data is uncorrelated with the noise. This results in the classic finite-length MMSE DFE design equations:

$$\begin{aligned} \boldsymbol{w}_{opt} &= [\boldsymbol{H}(\boldsymbol{I}_{L_c} - \boldsymbol{\Sigma}_\Delta^\top \boldsymbol{\Sigma}_\Delta)\boldsymbol{H}^\top + \sigma_n^2 \boldsymbol{I}_{L_w}]^{-1} \boldsymbol{H} \boldsymbol{e}_\Delta \\ \boldsymbol{g}_{opt} &= -\boldsymbol{\Sigma}_\Delta \boldsymbol{H}^\top \boldsymbol{w} \end{aligned}$$

We now turn to the design of $\tilde{\boldsymbol{g}}$, which is chosen so that μ taps of controlled ISI are left uncanceled by the DFSF. In [4] the μ non-zero taps in the effective channel were required to be contiguous, and consequently $\tilde{\boldsymbol{g}}$ was chosen to be equal to $\boldsymbol{g}$ but with the first $\mu-1$ taps set to zero. Here, since we are employing a BP detector, we have the flexibility of choosing the μ nonzero taps to appear anywhere in a non-contiguous fashion, giving us $\begin{pmatrix} L_c \\ \mu \end{pmatrix} = \frac{L_c!}{(L_c-\mu)!\mu!}$ possible choices. As such, the design of the DFSF involves two issues: picking the best locations for the μ nonzero taps in $\boldsymbol{c}$, and picking the values of the filter coefficients for best performance. As discussed in [13], large taps in the feedback portion of DFEs have a tendency to enhance the effects of error propagation. As such, we propose to zero the $\mu-1$ *largest* taps in $\boldsymbol{g}$ to create the filter $\tilde{\boldsymbol{g}}$ which has equal length. Thus, we leave the large taps in the effective impulse response $\boldsymbol{c}$ to be compensated by the more effective BP detector.

To illustrate the design of $\boldsymbol{w}$, $\boldsymbol{g}$ and $\tilde{\boldsymbol{g}}$, Fig. 2 provides an example showing the calculated impulse responses of the filters for the channel $\boldsymbol{h} = [0.5, 0.2, 1, 0.3]$ at 5 dB SNR, with $\Delta = 5$ and $\mu = 2$.

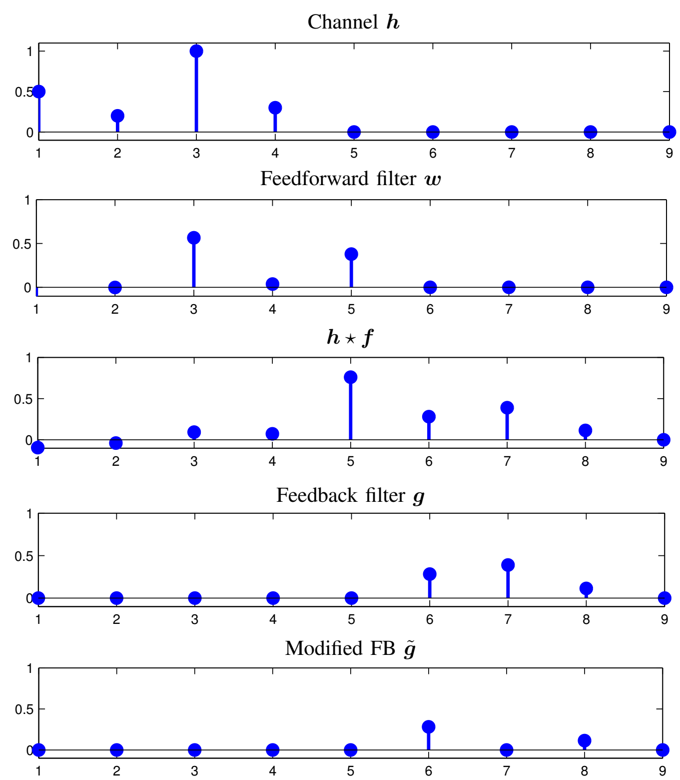

Fig. 2. Impulse responses

V. Noise Coloration

As mentioned previously, even if the noise $n[k]$ is white, the DFSF outputs colored noise. To see this, we let $\boldsymbol{W}$ be a wide Tœplitz convolution matrix corresponding to the filter $\boldsymbol{w}^\top$, and compute the covariance matrix of the noise observed by the BP detector as

$$\begin{aligned} \mathrm{E}\left\{\boldsymbol{v}\boldsymbol{v}^\top\right\} &= \mathrm{E}\left\{(\boldsymbol{W}\boldsymbol{n})(\boldsymbol{W}\boldsymbol{n})^\top\right\} = \boldsymbol{W}\mathrm{E}\left\{\boldsymbol{n}\boldsymbol{n}^\top\right\}\boldsymbol{W}^\top \\ &= \sigma_n^2 \boldsymbol{W}\boldsymbol{W}^\top \neq \sigma_n^2 \boldsymbol{I}. \end{aligned}$$

This is a problem because the BP algorithm assumes white noise. The coloration in the noise will harm the BP performance, potentially making it worse than a classical DFE or linear MMSE equalizer followed by a simple slicer. Thus, to avoid this pitfall, we consider a penalty term based on the squared autocorrelation of the output noise, or equivalently of the feedforward filter $\boldsymbol{w}$,

$$\begin{aligned} J_A(\boldsymbol{w}) &= \frac{1}{\|\boldsymbol{w}\|^4 \sigma_n^4} \sum_{l=1}^{L_w-1} \left|\mathrm{E}\left\{v_k v_{k-l}\right\}\right|^2 \\ &= \frac{1}{\|\boldsymbol{w}\|^4} \sum_{l=1}^{L_w-1} \left| \sum_{m=0}^{L_w-1} w_m w_{m-l} \right|^2 \\ &= \frac{1}{\|\boldsymbol{w}\|^4} \sum_{l=1}^{L_w-1} \sum_{m,n=0}^{L_w-1} w_m w_n w_{m-l} w_{n-l} \end{aligned} \quad (4)$$

It can be shown that J_A is equivalent to

$$J_A(\boldsymbol{w}) = \int_0^1 \left(\frac{|W(f)|^2}{\int_0^1 |W(f')|^2 \, df'} - 1 \right)^2 df + 1, \quad (5)$$

where $2\pi f = \omega$. Thus, J_A penalizes non-flatness of the spectrum of $\boldsymbol{w}$, since J_A drops to its minimum value of 1 as the spectrum $W(\omega)$ approaches any constant value $\forall f$.

In this paper, we explore the effect of choosing the DFSF taps to minimize the composite cost function given by

$$J(\boldsymbol{w}, \boldsymbol{g}) = J_{mse}(\boldsymbol{w}, \boldsymbol{g}) + \beta J_A(\boldsymbol{w}) \quad (6)$$

where β is a relative weighting term.

The value of β can be set several ways. The simplest is to try various values of β and get a sense of which values lead to good results for the class of parameter values of interest. For example, for the parameters in our preliminary simulations, $\beta \in [0.1, 0.5]$ seems to yield good results. Alternatively, β can be included in the optimization problem. One could search the objective function of (6) for a new value of $(\boldsymbol{w}, \boldsymbol{g})$ (but without changing β), then occasionally adjust β (but not $\boldsymbol{w}, \boldsymbol{g}$) to improve the BER, and repeat. If β is updated on a much slower time scale than $\boldsymbol{w}$ and $\boldsymbol{g}$, then the computationally-intensive BER does not have to be evaluated very often during the search.

For a given value of β, (6) can be minimized over $(\boldsymbol{w}, \boldsymbol{g})$ by any method of unconstrained non-linear optimization. We chose to use the simplex method of [14], since it was already available in Matlab, via the "fminsearch" function.

VI. Numerical Results

In order to provide evidence of the efficacy of the decision feedback sparsening filter design, we simulate two fading environments and compare their performance to Roy's sparsening filter design [2], a classical linear MMSE equalizer with a memoryless slicer and a classical DFE. In addition, to assess the performance degradation due to noise coloring, we include in the simulation a modified DFSF that minimizes the cost function given in (6), by adding the squared autocorrelation term that penalizes non-flatness of the spectrum of $\boldsymbol{w}$. To analyze the impact of the noise coloration in the DFSF scheme, we also compare it to the greedy Sparse Shortening SNR (SSSNR) [10] that also uses the squared autocorrelation penalizing term. We choose $\beta = 0.2$ for both schemes.

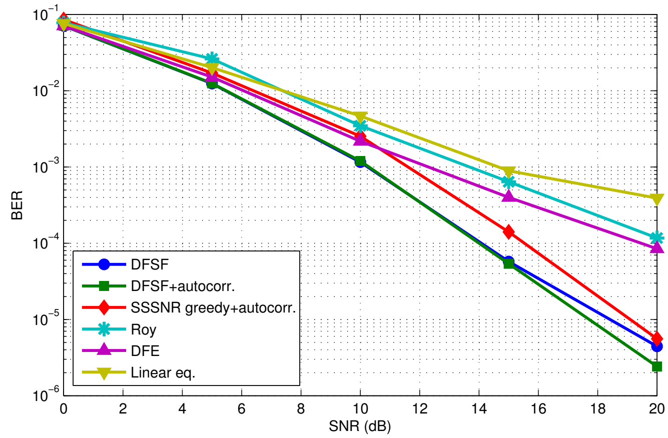

Fig. 3. Bit error rates for the Vehicular A channel

We first consider the ITU Vehicular A channel [15] that has six paths arriving at $[0, 310, 710, 1090, 1730, 2510]$ ns and a power-delay profile of $[0, -1, -9, -10, -15, -20]$ dB. In our simulations we used a square-root raised cosine pulse and a symbol duration of $T = 80$ns, which generally resulted in a sparse equivalent discrete channel with average length of 21 taps. Also, we transmit uncoded BPSK symbols and use 10 iterations in the BP detector. We design the DFSF to sparsen the channel to $\mu = 3$ taps, we let $L_w = 32$, $L_g = 40$ and the delay to be equal to $\Delta = 18$. For this simulation, the classical DFE also has $L_w = 32$ and $L_g = 40$ and the linear equalizer has length equal to 32.

The BER results are shown in Fig. 3. It is apparent that DFSF-based schemes employing a BP detector have the best performance among the simulated equalization methods. Notice that at a 10^{-4} BER the DFSF-based schemes outperform the classical DFE by approximately 4 dB, indicating a significant improvement in relation to Roy's and the other methods that do not employ the BP detector. When comparing to the greedy SSSNR design technique with the squared autocorrelation term, the improvement is about 1 dB at 10^{-5} BER. However, the greedy SSSNR algorithm requires the calculation of 93 sparsening filters, while the DFSF design requires the calculation of only one filter. This suggests that the DFSF design improves BER performance when compared to existing equalization-detection methods while reducing receiver complexity.

We also note that the use of the squared autocorrelation term does not provide great improvement in the DFSF BER performance for the vehicular channel. At 10^{-5} BER, the difference is only 0.5 dB. When comparing with the SSSNR greedy approach in [10] for the same channel, we verify that the difference between SSSNR greedy and SSSNR greedy+autocorrelation is approximately 2 dB before the satu-

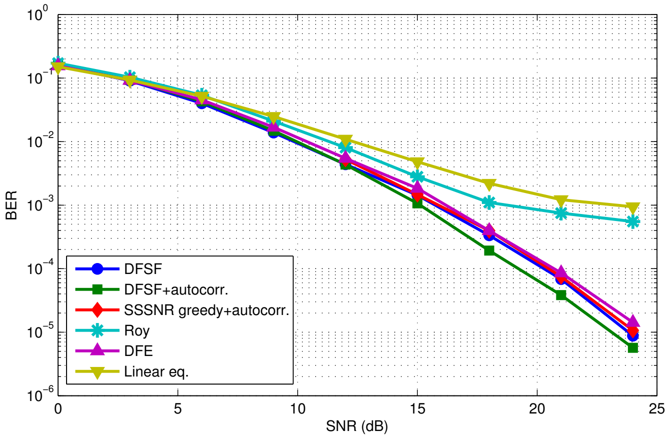

Fig. 4. Bit error rates for 5-tap channel with equal power delay profile

ration. This contrast arises from the fact that in the feedforward filter in the DFSF scheme already tends to be approximately flat for the vehicular channel and the addition of the penalizing term does not result in drastic noise-coloring improvement.

In addition, to show that the proposed scheme can also be employed successfully to mitigate non-sparse channels, we consider next a 5-tap fading channel with equal power delay profile of $[0.2, 0.2, 0.2, 0.2, 0.2]$. For this simulation environment, we transmit uncoded BPSK symbols, and we use 15 iterations in the BP detector. We design the DFSF to sparsen the channel to $\mu = 3$ non-zero taps, $L_w = 15$, $L_g = 5$ and the delay to be equal to $(\nu + L_w)/2$, where ν is the position of the largest tap in the channel. The length of the pre-filter for Roy's and SSSNR scheme is equal to 15 and both are also designed to use $\mu = 3$ non-zero taps. For this simulation, the classical DFE and the linear equalizer have $L_w = 15$ and the feedback filter for the DFE has length $L_g = 5$.

Fig. 4 shows the simulation results for the 5-tap fading channel with equal power delay profile. Once again the DFSF method with squared autocorrelation has the best performance among all simulated schemes. We also note that the addition of the squared autocorrelation term increased the performance improvement in relation to the Vehicular A channel.

VII. Conclusions

In this work we have considered the design of decision feedback sparsening filters as a way to reduce the complexity of iterative soft-input soft-output MAP detectors. By designing the sparsening filter so that the combined response of the (possibly non-sparse) channel and filter has a sparse impulse response, i.e. a response with only a handful of significant taps, the use of a BP-based MAP detector becomes feasible for detecting the bits. We proposed a filter design metric based on classical DFE design, but where we allow the largest $\mu - 1$ taps in the feedback filter to be left as residual ISI which is compensated by the BP detector. Numerical simulations showed that the proposed scheme can be employed satisfactorily to both sparse and non-sparse channels without requiring great computational cost in the receiver. Future work could investigate replacing tentative decisions in the feedback filter with more reliable decisions output by the BP detector.

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