Average Age of Information in Multi-Source Self-Preemptive Status Update Systems with Packet Delivery Errors

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***Abstract*—This paper studies the "age of information" (AoI) in a multi-source status update system where multiple sources send updates of their process to a monitor through a last-come first-served server with preemption in service and packet delivery errors. Arrival times of the status updates from the sources are assumed to be random according to independent Poisson processes. Service times are also assumed to be exponentially distributed and independent of the status arrivals. If the server is idle, any arriving packet immediately enters service. When the server is busy, if the arriving packet and the packet in service are from the same source, the packet in service is preempted and the new packet immediately enters service. Otherwise, any arriving packet is discarded. A closed-form expression for the average AoI of each source as a function of the system parameters is derived and, for the case without packet delivery errors, is compared to the average AoI in the "source agnostic" preemption setting considered by Yates and Kaul where any source can preempt any other source. The results show that source agnostic preemption in service results in better average AoI than self preemption in service for all sources.**

***Index Terms*—Age of information, multi-source, preemption, packet transmission error, stochastic hybrid systems.**

I. Introduction

In many networked monitoring and control systems, e.g., intelligent vehicular systems, timely status updates are critically important to maintain safe operation and provide stable control loops. This understanding has led to a new line of research centered around an *Age of Information* (AoI) metric which measures the staleness of a monitor's (or destination's) knowledge of a time varying process measured by a separate source in the network [1]–[5].

This paper analyzes the AoI in the multi-source single-destination status update system shown in Figure 1. The multi-source setting here has also been studied in [6]–[29] and is motivated by the recent study of a similar setting by Yates and Kaul in [30] where the AoI was studied under various assumptions about whether the server was first-come first-served (FCFS) or last-come last-served (LCFS) and, for the latter case, whether new updates could preempt packets currently in service or only in waiting. A key assumption in [30], however, is that preemption is "source agnostic" in the sense that packets from any source can preempt packets from any other source. This assumption, at least intuitively, seems to penalize sources that send updates relatively infrequently since those sources are unlikely to be able to successfully deliver a packet before it is preempted by sources that send more frequent updates. This assumption may also not be practical in settings where sources are prevented from preempting other sources.

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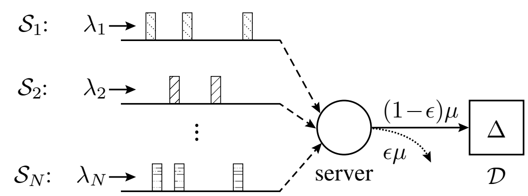

Fig. 1. The multi-source status update system with unreliable transmissions. Every packet that is preempted and also packets that find the server busy but cannot preempt the packet in service, are discarded. Status updates arrive at source i with rate λ_i based on Poisson point process. Packets in service depart the server (complete service) with rate μ if not preempted or lost.

Motivated by these considerations, this paper studies an LCFS system with the key assumption that source $\mathcal{S}_i$ can only preempt its own packets in service. To distinguish this approach from the approach in [30], we refer to this as "self preemption" and the model of Yates and Kaul as "global preemption". We further generalize the model by allowing for transmission errors from the server with fixed probability $0 \leq \epsilon < 1$. Using tools from stochastic hybrid systems (SHS), we derive a closed-form expression for average AoI experienced by each source in terms of the status update arrival rates $\{\lambda_1, \dots, \lambda_N\}$, service rate μ, and the transmission error probability ϵ. We show, somewhat surprisingly, that global preemption in service (referred to as LCFS-S in [30]) provides uniformly better AoI than self preemption in service for all sources in the system when $\epsilon = 0$. Numerical examples are also presented to quantify the average AoI and verify the analysis.

II. System Model

We consider a status update system with N source nodes $\mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_N$ and one destination node $\mathcal{D}$ as represented in Figure 1. Source $\mathcal{S}_i$ generates packets containing status updates at successive times based on a Poisson point process with rate λ_i independently of the other sources and the service times of the server. A server with service rate μ according to the exponential distribution delivers packets to the destination.

We assume that a packet in service has a transmission error (is not delivered) with fixed probability $0 \leq \epsilon < 1$.

Packets containing status updates from source $\mathcal{S}_i$ immediately enter service if (i) the server is idle or (ii) a packet from source $\mathcal{S}_i$ is currently in service. In the latter case, the packet currently in service is dropped and the new packet enters service. If a packet from source $\mathcal{S}_i$ is in service and a new packet from source $\mathcal{S}_j$ with $j \neq i$ arrives at the server, the new packet from source $\mathcal{S}_j$ is discarded.

For notational convenience, we define the normalized rates

$$\rho_i = \frac{\lambda_i}{\mu}, \tag{1a}$$

$$\rho = \sum_{i=1}^{N} \rho_i, \text{ and} \tag{1b}$$

$$\rho_{-i} = \rho - \rho_i \tag{1c}$$

for $i \in \{1, 2, \ldots, N\}$, where ρ_i represents the offered load of source $\mathcal{S}_i$ and ρ represents the total offered load [2].

A. Average Age Metric

The age $\Delta_i(t)$ of the status updates of source $\mathcal{S}_i$ at the destination is a linearly increasing random process when no updates arrive at the destination and has downward jumps when an update completes service. The age of some information generated at time t' at the source node $\mathcal{S}_i$ and observed at the destination at time t, where $t \geq t'$, is defined as the time elapsed between t' and t, i.e., $\Delta_i(t) \triangleq t - t'$.

The average age of the status updates of source $\mathcal{S}_i$ at the destination is equal to the area under $\Delta_i(t)$ divided by the observation interval. Over an observation interval $(\bar{t}, \bar{t} + \mathcal{T})$, where $\bar{t}$ is such that at least one status update has been received from source $\mathcal{S}_i$, the average age is defined as

$$\Delta_i(\bar{t}, \bar{t} + \mathcal{T}) \triangleq \frac{1}{\mathcal{T}} \int_{\bar{t}}^{\bar{t}+\mathcal{T}} \Delta_i(t) \, dt. \tag{2}$$

Letting the observation interval become large, the average age of the state information of source $\mathcal{S}_i$ from the perspective of the destination is [2]

$$\Delta_i \triangleq \lim_{\mathcal{T} \to \infty} \Delta_i(\bar{t}, \bar{t} + \mathcal{T}). \tag{3}$$

III. Average Age of Information Analysis

Theorem 1 provides an expression for the average age of information from source $\mathcal{S}_i$ in the status update system described in Section II.

Theorem 1. *The average age of information Δ_i of the status updates of source $i \in \{1, 2, \ldots, N\}$ for the multi-source system with self preemption in service and service completion with error is equal to*

$$\Delta_i = \frac{\rho_i^3 + \rho_i^2(2\rho_{-i}+3) + \rho_i[\rho_{-i}^2 + \rho_{-i}(5-\epsilon)+3] + (\rho_{-i}+1)^2}{\mu(1-\epsilon)\rho_i(\rho_i+1)(\rho+1)}. \tag{4}$$

Proof:

The proof follows the stochastic hybrid systems (SHS) framework first proposed for average AoI analysis in [31]. A Markov chain representation of state $q(t) \in \mathcal{Q}$ of the system from the perspective of source $\mathcal{S}_1$ is shown in Figure 2. The states are indexed by $q \in \mathcal{Q} = \{0, 1, \ldots, N\}$, where state 0 indicates the server is idle and state $q \geq 1$ indicates a packet from source q is in service.

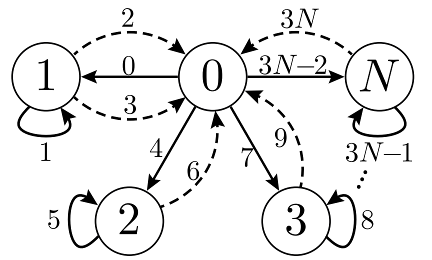

Fig. 2. The Markov chain representation of the multi-source status update system in Figure 1 from the perspective of source $\mathcal{S}_1$. States are indexed by $q \in \mathcal{Q} = \{0, \ldots, N\}$. Packet arrivals and service completions are represented by solid and dashed arrows, respectively.

In Figure 2, link 0 corresponds to a packet arriving from source $\mathcal{S}_1$ when the server is idle. Link 1 corresponds to a packet arriving from source $\mathcal{S}_1$ when the server is currently serving a packet from source $\mathcal{S}_1$ (self preemption in service). Link 2 corresponds to a successfully delivered packet and link 3 corresponds to a transmission error. Note that these links are also present for sources $\{\mathcal{S}_2, \ldots, \mathcal{S}_N\}$, but successful and unsuccessful deliveries are lumped into single links to simplify the analysis of the AoI of source $\mathcal{S}_1$. Also note that our self-preemption assumption prevents us from lumping $\{\mathcal{S}_2, \ldots, \mathcal{S}_N\}$ as a single "effective" source as in [30].

From the SHS method, without loss of generality let $\mathbf{x}(t) = [x_0(t), x_1(t)]$ denote the continuous state vector, where $x_0(t)$ is the current age $\Delta_1(t)$ and $x_1(t)$ is the reduction in $\Delta_1(t)$ that will occur when the packet in service is delivered. Table I represents the exponential rates at which state $q(t^-)$ transitions to $q'(t) = q(t^+)$ in the Markov chain in Figure 2 and the transition map $\phi(q(t^-), \mathbf{x}(t^-)) = (q'(t), \mathbf{x}'(t)) = (q(t^+), \mathbf{x}(t^+))$ for each link ℓ. For example, the occurrence of link $\ell = 0$ at time t shows that the system is empty at t^-; then, a packet arrives from source 1 and the system transitions to state $q(t^+) = 1$. When this transition occurs, we have $x_0'(t) = x_0(t^+) = x_0(t^-) = x_0(t)$, and when this packet completes service $\Delta_1(t)$ drops by $x_0(t)$, giving $x_1'(t) = x_0(t^+)$. For notational simplicity, denote $x_0 = x_0(t)$ and $x_1 = x_1(t)$.

TABLE I
Transition rates for the Markov chain, $i = \{2, \ldots, N\}$.

link ℓ	$q \to q'$	rate $\lambda^{(\ell)}$	$\phi(q, x) = (q', x')$
0	$0 \to 1$	$\lambda_1 \delta_{0,q}$	$(1, [x_0, x_0])$
1	$1 \to 1$	$\lambda_1 \delta_{1,q}$	$(1, [x_0, x_0])$
2	$1 \to 0$	$(1-\epsilon)\mu\delta_{1,q}$	$(0, [x_0 - x_1, 0])$
3	$1 \to 0$	$\epsilon\mu\delta_{1,q}$	$(0, [x_0, 0])$
$3(i-1)+1$	$0 \to i$	$\lambda_i \delta_{0,q}$	$(i, [x_0, 0])$
$3(i-1)+2$	$i \to i$	$\lambda_i \delta_{i,q}$	$(i, [x_0, 0])$
$3i$	$i \to 0$	$\mu \delta_{i,q}$	$(0, [x_0, 0])$

The SHS method uses test functions whose expected values converge to steady-state quantities of interest, such as the average age [30]. For $\bar{q} \in \mathcal{Q}$ and binary vectors $m = (m_0, m_1)$, the system will employ the test functions

$$\psi_{\bar{q}}^{(m)} = \mathbf{x}^m \delta_{\bar{q},q}, \tag{5}$$

where $\delta_{\bar{q},q}$ denotes the Kronecker delta function and $\mathbf{x}^m$ is shorthand for the monomial $\mathbf{x}^m = [x_0, x_1]^m = x_0^{m_0} x_1^{m_1}$. For $m = (0,0)$, the expected value of the test function $\psi_{\bar{q}}^{(0,0)}(q, \mathbf{x})$ gives the steady state probabilities

$$\pi_{\bar{q}}^*(t) = P[q(t) = \bar{q}] = E[\delta_{\bar{q},q(t)}] = E[\psi_{\bar{q}}^{(0,0)}(q(t), \mathbf{x}(t))]. \tag{6}$$

Associated with the SHS is a mapping $\Psi \to L\Psi$ given by

$$(L\psi)(q, \mathbf{x}) = \frac{\partial \psi(q, \mathbf{x})}{\partial x_0} + \sum_{\ell \in \mathcal{L}} (\psi(\phi(q, \mathbf{x})) - \psi(q, \mathbf{x}))\lambda^{(\ell)}(q), \tag{7}$$

where $\mathcal{L}$ represents the set of all links in the Markov chain. Each test function must satisfy

$$\frac{dE[\psi(q(t), \mathbf{x}(t))]}{dt} = E[(L\psi)(q(t), \mathbf{x}(t))]. \tag{8}$$

We define the corresponding variables

$$v_{0,\bar{q}}(t) = E[\psi_{\bar{q}}^{(1,0)}(q(t), \mathbf{x}(t))] = E[x_0(t)\delta_{\bar{q},q(t)}], \tag{9a}$$
$$v_{1,\bar{q}}(t) = E[\psi_{\bar{q}}^{(0,1)}(q(t), \mathbf{x}(t))] = E[x_1(t)\delta_{\bar{q},q(t)}]. \tag{9b}$$

The SHS method applies (7) and (8) to each function $\psi(q, \mathbf{x}) = \psi_{\bar{q}}^{(m)}(q, \mathbf{x})$. This gives a set of first order linear differential equations for $\pi_{\bar{q}}(t)$ and $v_{i\bar{q}}(t)$ for $\bar{q} \in \mathcal{Q}$ and $i = \{0, 1\}$. We will obtain a number of equations as follows. From (7) we calculate $L\psi_{\bar{q}}^{(m)}$ for each m and $\mathcal{Q}$ as

$$L\psi_{\bar{q}}^{(m)}(q, \mathbf{x}) = m_0 x_0^{m_0 - 1} x_1^{m_1} \delta_{\bar{q},q} + \mu \Lambda_{\bar{q}}^{(m)}(q, \mathbf{x}), \tag{10}$$

where $\Lambda_{\bar{q}}^{(m)}(q, \mathbf{x})$ is shortened for $\Lambda_{\bar{q}}^{(m)}(q(t), \mathbf{x}(t))$ and

$$\Lambda_{\bar{q}}^{(m)}(q, \mathbf{x}) = \frac{1}{\mu} \sum_{\ell \in \mathcal{L}} (\psi(\phi(q, \mathbf{x})) - \psi(q, \mathbf{x}))\lambda^{(\ell)}(q). \tag{11}$$

Considering (11) we can write

$$\Lambda_0^{(m)} = -\rho \mathbf{x}^m \delta_{0,q} + (1-\epsilon)[x_0 - x_1, 0]^m \delta_{1,q} + \epsilon [x_0, 0]^m \delta_{1,q} + [x_0, 0]^m (\delta_{2,q} + \delta_{3,q} \ldots + \delta_{N,q}), \tag{12a}$$
$$\Lambda_1^{(m)} = \rho_1 [x_0, x_0]^m \delta_{0,q} - (\rho_1 + 1)\mathbf{x}^m \delta_{1,q} + \rho_1 [x_0, x_0]^m \delta_{1,q}, \tag{12b}$$
$$\Lambda_{\bar{q}}^{(m)} = \rho_i [x_0, 0]^m \delta_{0,q} - (\rho_i + 1)\mathbf{x}^m \delta_{i,q} + \rho_i [x_0, 0]^m \delta_{i,q}, \tag{12c}$$

for $\bar{q} \in \{2, 3, \ldots, N\}$. By applying (8) to $\psi(q(t), \mathbf{x}(t)) = \psi_{\bar{q}}^{(m)}(q(t), \mathbf{x}(t))$, it follows from (10) that

$$\frac{dE[\psi_{\bar{q}}^{(m)}(q(t), \mathbf{x}(t))]}{dt} = m_0 E[\mathbf{x}^{(m_0 - 1, m_1)}(t)\delta_{\bar{q},q(t)}] + \mu E[\Lambda_{\bar{q}}^{(m)}(q(t), \mathbf{x}(t))]. \tag{13}$$

From (6) and (11) we get $\dot{\pi}_{\bar{q}}(t) = \mu \mathbf{R}\pi(t)$. Setting $\dot{\pi}(t) = 0$ yields the steady state probabilities π_i^* that satisfy $\mathbf{R}\pi^* = 0$ and $\sum_{i=0}^N \pi_i^* = 1$, giving

$$\pi_0^* = \frac{1}{\rho + 1}, \quad \pi_i^* = \frac{\rho_i}{\rho + 1}, \quad i \in \{1, 2, \ldots, N\}. \tag{14a}$$

For $m \in \{(1,0), (0,1)\}$ from (9a)-(9b) and (13) we get

$$\dot{v}_{0,\bar{q}}(t) = E[\delta_{\bar{q},q(t)}] + \mu E[\Lambda_{\bar{q}}^{(1,0)}(q(t), \mathbf{x}(t))], \tag{15a}$$
$$\dot{v}_{1,\bar{q}}(t) = \mu E[\Lambda_{\bar{q}}^{(0,1)}(q(t), \mathbf{x}(t))]. \tag{15b}$$

From (15a)-(15b) we get

$$\dot{v}_{0,0}(t) = \mu[-\rho v_{0,0} + v_{0,1} + \ldots + v_{0,N} - (1-\epsilon)v_{1,1}] + \pi_0^*, \tag{16a}$$
$$\dot{v}_{0,1}(t) = \mu[\rho_1 v_{0,0} - v_{0,1}] + \pi_1^*, \tag{16b}$$
$$\dot{v}_{0,i}(t) = \mu[\rho_i v_{0,0} - v_{0,i}] + \pi_i^*, \tag{16c}$$
$$\dot{v}_{1,0}(t) = -\rho v_{1,0}, \tag{16d}$$
$$\dot{v}_{1,1}(t) = \rho_1 v_{0,0} + \rho_1 v_{0,1} - (\rho_1 + 1)v_{1,1}, \tag{16e}$$
$$\dot{v}_{1,i}(t) = -(\rho_i + 1)v_{1,i}, \tag{16f}$$

for $i \in \{2, 3, \ldots, N\}$. We gather the nontrivial variables in the following two vectors

$$\mathbf{v}_0 = \begin{bmatrix} v_{0,0} & v_{0,1} & v_{0,2} & \ldots & v_{0,N} \end{bmatrix}', \quad \mathbf{v}_1 = \begin{bmatrix} v_{1,1} \end{bmatrix}'. \tag{17a}$$

Setting $\dot{\mathbf{v}}_0 = 0$ and $\dot{\mathbf{v}}_1 = 0$ and solving for the nontrivial variables, the average age Δ is obtained as

$$\Delta = \sum_{\bar{q} \in \mathcal{Q}} v_{0,\bar{q}}^* \tag{18}$$

where $v_{0,\bar{q}}^* = \lim_{t \to \infty} v_{0,\bar{q}}(t)$. Solving the differential equations for variables $v_{0,\bar{q}}(t)$, we get

$$v_{0,0}^* = \frac{\rho_1^2 \epsilon + \rho_1(2 + \rho_{-1}) + \rho_{-1} + 1}{\mu(1-\epsilon)\rho_1(\rho_1 + 1)(\rho + 1)}, \tag{19a}$$
$$v_{0,i}^* = \frac{\rho_i[\rho_1^2 + \rho_1(\rho_{-1} - \epsilon + 3) + \rho_{-1} + 1]}{\mu(1-\epsilon)\rho_1(\rho_1 + 1)(\rho + 1)}, \tag{19b}$$

for $i \in \{1, 2, \ldots, N\}$. From (18) and (19a)-(19b) we get

$$\Delta_1 = \frac{\rho_1^3 + \rho_1^2(2\rho_{-1} + 3) + \rho_1[\rho_{-1}^2 + \rho_{-1}(5 - \epsilon) + 3] + (\rho_{-1} + 1)^2}{\mu(1-\epsilon)\rho_1(\rho_1 + 1)(\rho + 1)}. \tag{20}$$

Because of the symmetry of the problem, the average AoI of source i is obtained as in (4), which completes the proof. □

In the absence of packet delivery errors ($\epsilon = 0$), the following Corollary compares self preemption and global preemption in service (LCFS-S in [30]).

Corollary 1. *In the absence of packet delivery errors, i.e., $\epsilon = 0$, global preemption in service has a lower average AoI than self preemption in service for all sources.*

Proof: From Theorem 2(a) of [30], we have

$$\Delta_{i,\text{glob}} = \frac{1}{\mu}(1 + \rho)\frac{1}{\rho_i}. \tag{21}$$

Subtracting this from (4) results in

$$\Delta_{i,\text{self}} - \Delta_{i,\text{glob}} = \frac{\rho_{-i}}{\mu(\rho_i + 1)(\rho + 1)} \geq 0, \tag{22}$$

since all of the system parameters are non-negative. □

Finally, considering the average AoI in (4), for $\epsilon = 0$ and $\rho_i \to \rho$, we have

$$\lim_{\rho_i \to \rho} \Delta_i = \frac{1}{\mu\rho}(1 + \rho) = \frac{1}{\lambda} + \frac{1}{\mu} \tag{23}$$

which is identical to the average AoI of a single-source M/M/1 system with LCFS discipline and preemption in service [32].

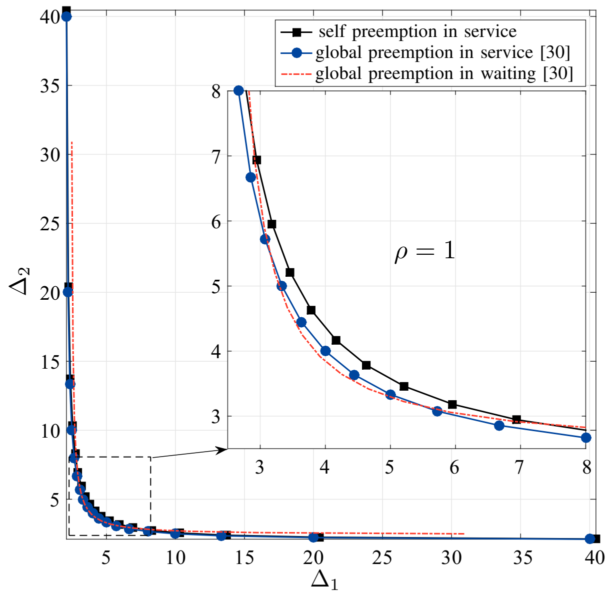

Fig. 3. Comparison of the achievable age pairs of the proposed scenario with self preemption in service with the global preemption in service and global preemption in waiting cases for $\mu = 1$, $\rho = 1$, and $\epsilon = 0$.

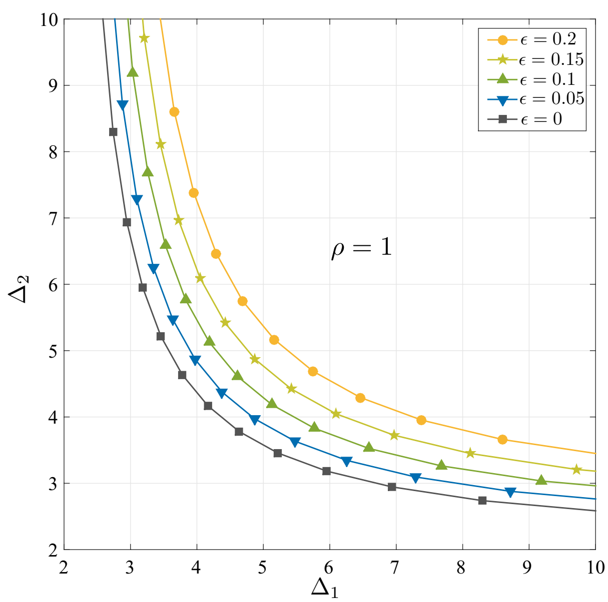

Fig. 4. Achievable age pairs of the proposed scenario with self preemption in service for $\mu = 1$, $\rho = 1$, and $\epsilon \in \{0, 0.05, 0.1, 0.15, 0.2\}$.

IV. Numerical Results

This section presents numerical examples to illustrate the achieved average AoI under various system parameters. All of the results assume a normalized service rate of $\mu = 1$.

We first consider a status update system with $N = 2$ sources with a fixed total packet arrival rate $\rho = 1$ and no packet delivery errors ($\epsilon = 0$). Figure 3 shows the average AoI pairs (Δ_1, Δ_2) of systems with self preemption in service, global preemption in service, and global preemption in waiting. As expected from Corollary 1, global preemption in service uniformly outperforms self preemption in service. The results also show that when $\rho_1 \to 1$, the average AoI of $\mathcal{S}_1$ for the case with self preemption in service converges to the result in (23) and approaches the average AoI of a single-source M/M/1 system with LCFS discipline and preemption in service.

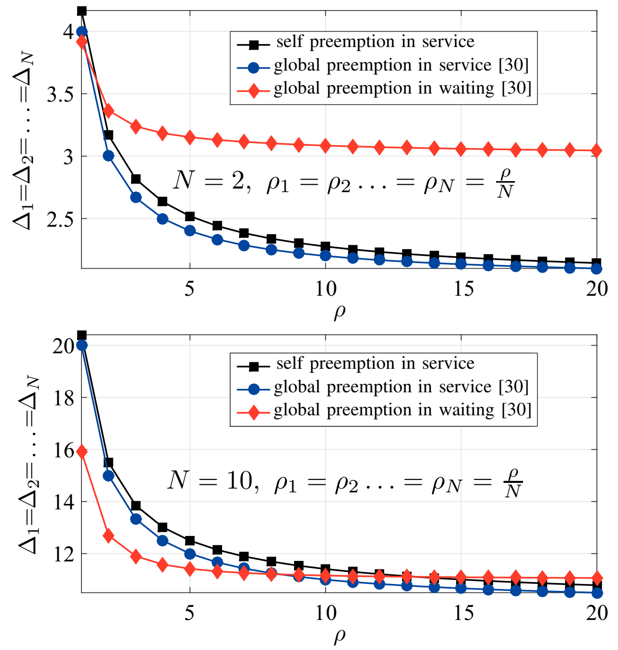

Fig. 5. Comparison of the achievable average AoI vs ρ where $\rho_1 = \rho_2 = \ldots = \rho_N = \frac{\rho}{N}$ for $\mu = 1$, $\epsilon = 0$, and $N \in \{2, 10\}$.

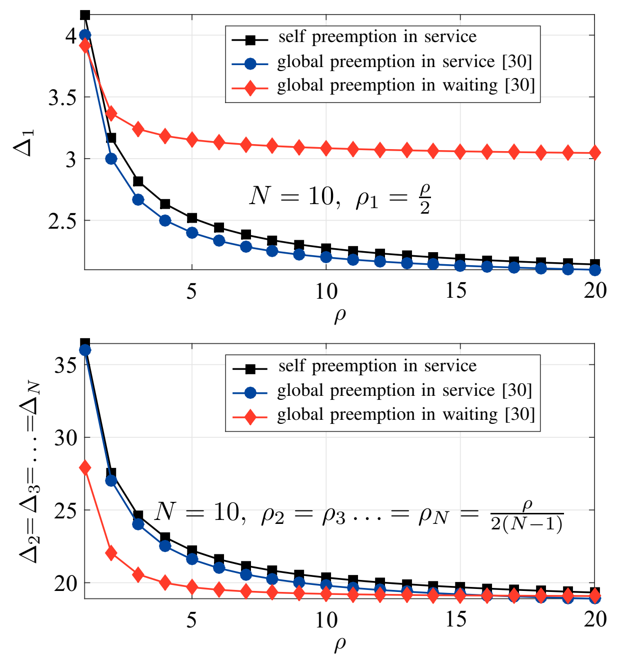

Fig. 6. Comparison of the achievable average AoI vs ρ where $\rho_1 = \frac{\rho}{2}$, $\rho_2 = \rho_3 = \ldots = \rho_N = \frac{\rho}{2(N-1)}$ for $\mu = 1$, $\epsilon = 0$, and $N = 10$.

Figure 4 considers the same setting except with non-zero packet delivery error probabilities. The results show that the average AoI pairs strictly increase with ϵ. Intuitively, the average number of packets successfully delivered to the destination over any interval decreases as ϵ increases.

We next consider a system with symmetric loading, i.e.,

$\rho_i = \frac{\rho}{N}$ for all $i \in \{1,\ldots,N\}$, and no packet delivery errors. Figure 5 plots the average AoI versus the total packet arrival rate ρ for $N \in \{2, 10\}$. The results show that average AoI is decreasing in ρ for all three preemption schemes and that global preemption in service uniformly outperforms self preemption in service. For small values of ρ, global preemption in waiting can outperform either preemption in service discipline.

Finally, we consider a system with asymmetric loading, i.e., $\rho_1 = \frac{\rho}{2}$ and $\rho_i = \frac{\rho}{2(N-1)}$ for $i \in \{2, \ldots, N\}$, and no packet delivery errors. Figure 6 represents the achieved average AoI for the three cases versus different total packet arrival rate ρ for $N = 10$. The results show that the average AoI for source $\mathcal{S}_1$ is identical to the symmetric setting with $N = 2$ since source $\mathcal{S}_1$ represents half of the load to the server. The average AoI for the remaining sources is worse than the symmetric setting with $N = 10$ due to each source $i \in \{2, \ldots, 10\}$ receiving smaller fraction of the total load than in the symmetric case.

V. Conclusion

This paper studied the AoI problem in a multi-source status update system with transmission errors. The server allows preemption of the packets in service only by newly-arriving packets from the same source. An average AoI expression was derived as a function of the system parameters in this setting and, for the case without transmission errors, was compared to the global preemption in service setting of Yates and Kaul. We showed, somewhat surprisingly, that global preemption in service results in better average AoI than self preemption in service *for all sources*. Future directions of this work include generalizations of the model to a setting with multiple servers.

References

[1] S. Kaul, M. Gruteser, V. Rai, and J. Kenney, "Minimizing age of information in vehicular networks," in *Proc. IEEE SECON*, Jun. 2011, pp. 350–358.

[2] S. Kaul, R. Yates, and M. Gruteser, "Real-time status: How often should one update?" in *Proc. IEEE INFOCOM*, Mar. 2012, pp. 2731–2735.

[3] Y. Sun, E. Uysal-Biyikoglu, R. D. Yates, C. E. Koksal, and N. B. Shroff, "Update or wait: How to keep your data fresh," *IEEE Trans. Inf. Theory*, vol. 63, no. 11, pp. 7492–7508, Aug. 2017.

[4] S. Farazi, A. G. Klein, and D. R. Brown III, "Average age of information for status update systems with an energy harvesting server," in *Proc. IEEE Intl. Conf. on Computer Comm. Workshops (INFOCOM WKSHPS)*, Apr. 2018, pp. 112–117.

[5] S. Farazi, A. G. Klein, and D. R. Brown III, "Age of information in energy harvesting status update systems: When to preempt in service?" in *Proc. IEEE Intl. Symp. on Inf. Theory (ISIT)*, Jun. 2018, pp. 2436–2440.

[6] L. Huang and E. Modiano, "Optimizing age-of-information in a multi-class queueing system," in *Proc. IEEE Intl. Symp. on Inf. Theory (ISIT)*, Jun. 2015, pp. 1681–1685.

[7] S. Farazi, D. R. Brown III, and A. G. Klein, "On global channel state estimation and dissemination in ring networks," in *Proc. Asilomar Conf. on Signals, Systems, and Computers*, Nov. 2016, pp. 1122–1127.

[8] S. Farazi, A. G. Klein, and D. R. Brown III, "On the average staleness of global channel state information in wireless networks with random transmit node selection," in *Proc. IEEE Intl. Conf. on Acoustics, Speech and Signal Processing (ICASSP)*, Mar. 2016, pp. 3621–3625.

[9] Q. He, D. Yuan, and A. Ephremides, "On optimal link scheduling with min-max peak age of information in wireless systems," in *Proc. IEEE Intl. Conf. on Comm. (ICC)*, May 2016, pp. 1–7.

[10] I. Kadota, E. Uysal-Biyikoglu, R. Singh, and E. Modiano, "Minimizing the age of information in broadcast wireless networks," in *Proc. of Allerton Conf. on Commun., Contr., and Computing*, Sep. 2016, pp. 844–851.

[11] V. Tripathi and S. Moharir, "Age of information in multi-source systems," in *Proc. IEEE Global Telecomm. Conf. (GLOBECOM)*, Dec. 2017, pp. 1–6.

[12] A. G. Klein, S. Farazi, W. He, and D. R. Brown III, "Staleness bounds and efficient protocols for dissemination of global channel state information," *IEEE Trans. Wireless Commun.*, vol. 16, no. 9, pp. 5732–5746, Sep. 2017.

[13] R. D. Yates and S. K. Kaul, "Status updates over unreliable multiaccess channels," in *Proc. IEEE Intl. Symp. on Inf. Theory (ISIT)*, Aug. 2017, pp. 331–335.

[14] S. Farazi, A. G. Klein, and D. R. Brown III, "Bounds on the age of information for global channel state dissemination in fully-connected networks," in *Proc. Intl. Conf. on Computer Communication and Networks (ICCCN)*, Jul. 2017, pp. 1–7.

[15] R. Talak, S. Karaman, and E. Modiano, "Minimizing age-of-information in multi-hop wireless networks," in *Proc. of Allerton Conf. on Commun., Contr., and Computing*, Oct. 2017.

[16] Q. He, D. Yuan, and A. Ephremides, "Optimal link scheduling for age minimization in wireless systems," *IEEE Trans. Inf. Theory*, vol. 64, no. 7, pp. 5381–5394, Jul. 2018.

[17] S. Farazi, A. G. Klein, J. A. McNeill, and D. R. Brown III, "On the age of information in multi-source multi-hop wireless status update networks," in *Proc. IEEE SPAWC*, Jun. 2018.

[18] E. Najm and E. Telatar, "Status updates in a multi-stream m/g/1/1 preemptive queue," in *Proc. IEEE Intl. Conf. on Computer Comm. Workshops (INFOCOM WKSHPS)*. IEEE, 2018, pp. 124–129.

[19] R. Talak, I. Kadota, S. Karaman, and E. Modiano, "Scheduling policies for age minimization in wireless networks with unknown channel state," in *Proc. IEEE Intl. Symp. on Inf. Theory (ISIT)*, Jun. 2018, pp. 2564–2568.

[20] Z. Jiang, B. Krishnamachari, X. Zheng, S. Zhou, and Z. Niu, "Decentralized status update for age of information optimization in wireless multiaccess channels," in *Proc. IEEE Intl. Symp. on Inf. Theory (ISIT)*, Jun. 2018, pp. 2276–2280.

[21] Y. Sun, E. Uysal-Biyikoglu, and S. Kompella, "Age-optimal updates of multiple information flows," in *Proc. IEEE Intl. Conf. on Computer Comm. Workshops (INFOCOM WKSHPS)*. IEEE, 2018, pp. 136–141.

[22] R. Talak, S. Karaman, and E. Modiano, "Distributed scheduling algorithms for optimizing information freshness in wireless networks," in *Proc. IEEE SPAWC*. IEEE, 2018, pp. 1–5.

[23] Z. Jiang, B. Krishnamachari, S. Zhou, and Z. Niu, "Can decentralized status update achieve universally near-optimal age-of-information in wireless multiaccess channels?" in *Intl. Teletraffic Congress (ITC 30)*, vol. 1. IEEE, 2018, pp. 144–152.

[24] Y.-P. Hsu, "Age of information: Whittle index for scheduling stochastic arrivals," *arXiv preprint arXiv:1801.03422*, 2018.

[25] S. K. Kaul and R. D. Yates, "Age of information: Updates with priority," in *Proc. IEEE Intl. Symp. on Inf. Theory (ISIT)*, Jun. 2018, pp. 2644–2648.

[26] S. Farazi, A. G. Klein, and D. R. Brown III, "Fundamental bounds on the age of information in multi-hop global status update networks," *Journal of Communications and Networks*, vol. 21, no. 3, pp. 268–279, 2019.

[27] A. Kosta, N. Pappas, A. Ephremides, and V. Angelakis, "Age of information performance of multiaccess strategies with packet management," *Journal of Communications and Networks*, vol. 21, no. 3, pp. 244–255, 2019.

[28] S. Farazi, A. G. Klein, and D. R. Brown III, "Fundamental bounds on the age of information in general multi-hop interference networks," in *Proc. IEEE Intl. Conf. on Computer Comm. Workshops (INFOCOM WKSHPS)*, Apr. 2019, pp. 96–101.

[29] E. Najm, R. Nasser, and E. Telatar, "Content based status updates," *IEEE Transactions on Information Theory*, 2019.

[30] R. D. Yates and S. K. Kaul, "The age of information: Real-time status updating by multiple sources," *IEEE Transactions on Information Theory*, vol. 65, no. 3, pp. 1807–1827, 2019.

[31] ——, "The age of information: Real-time status updating by multiple sources," *arXiv preprint arXiv:1608.08622v1*, 2016.

[32] S. K. Kaul, R. D. Yates, and M. Gruteser, "Status updates through queues," in *Proc. Conference on Inf. Sciences and Systems (CISS)*, Mar. 2012, pp. 1–6.