Relay Selection in Amplify-and-Forward Relay Networks with Frequency Selective Fading

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***Abstract*—We consider relay selection in cooperative relay networks with frequency selective fading, and focus on a system where multiple amplify-and-forward relays share a single channel orthogonal to the source. We propose a relay selection method that achieves the optimal diversity-multiplexing tradeoff (DMT) as proven by outage analysis. This relay selection method, combined with zero-padded transmission and maximum likelihood sequential estimation (MLSE), can achieve full diversity as corroborated by numerical results. Since MLSE may not be affordable in practice due to its high complexity, another relay selection method based on the average decision-point SNR is proposed for linear zero-forcing equalization at the destination, and it asymptotically achieves the optimal DMT.**

Review topics: C. Networks 4. Cooperative Diversity

I. Introduction

Cooperative relay networks have emerged as a powerful technique to combat multipath fading and increase energy efficiency. Compared to systems employing multiple orthogonal relay subchannels with repetition coding and distributed space-time/space-frequency codes, simpler relay selection (RS) can achieve good spectral efficiency and cooperative diversity without requiring strict synchronization or high decoding complexity. However, RS methods have predominantly focused on the frequency flat fading channels [1], [2]. RS in the presence of frequency selective (FS) fading has seen some attention in OFDM systems [3], [4], [5]. For example, in [3], [4], an uncoded OFDM system is studied, and it was shown that if relay selection is done on a per-subcarrier basis, full spatial diversity can be achieved. However, neither of these OFDM-based relay selection methods were able to exploit the frequency diversity. A linearly precoded OFDM system was proposed in [5] which uses multiple amplify-forward relays with linear transmit precoding; a simulation-based study showed that two relay selection schemes exhibited a coding gain improvement compared to an orthogonal round-robbin relaying scheme. None of them present analytical results on RS with full diversity. We presented analytical results of decode-and-forward (DF) relay selection in [6] where we proposed a relay selection method which can attain full diversity. In this paper, we consider FS fading and amplify-and-forward (AF) half-duplex relays, and we propose relay selection strategies and transceiver structures for single-carrier transmission that achieve the optimal diversity-multiplexing tradeoff (DMT) and therefore full diversity.

II. System Model

We consider a system as in Fig. 1, which consists of a single source node ($\mathbf{S}$), K relay nodes ($\mathbf{R}_{1,2,\ldots,K}$), and a single destination node ($\mathbf{D}$). We assume that all nodes have the same average power constraint P watts and transmission bandwidth W Hz. While this model has been well-studied in the case of static flat channels [1], here the links between the nodes are assumed to be quasi-static FS fading channels. Each fading tap of the channel $\boldsymbol{h}_{jk} \in \mathbb{C}^{L_{jk}}$ is i.i.d. Rayleigh fading with variance 1 with jk as sd, sr_i or $r_i d$. We assume perfect channel state information (CSI) at the destination and no CSI at the source. In addition, all links have additive noise which is assumed to be mutually independent, zero-mean circularly symmetric complex Gaussian with variance N_0 and the discrete-time SNR is defined as $\rho \triangleq \frac{P}{WN_0}$. Using a combination of the cut-set bound and matched filter bound (MFB), we recently showed in [6] that the optimal DMT of the channel model in Fig. 1 is

$$d(r) \leq (L_{sd} + \sum_{i=1}^{K} \min(L_{sr_i}, L_{r_i d}))(1-2r), \qquad (1)$$

which is also referred to as the upper bound on DMT. It is the benchmark against which we will compare the performance of the two proposed RS methods we will show next. One employs maximum likelihood sequential estimation (MLSE) and the other employs linear zero-forcing equalization (ZFE). As we will see, while MLSE has higher complexity in equalization, the corresponding RS method does not require much computation; on the other hand, linear ZFE has lower complexity in equalization, but the selection method designed to achieve the optimal DMT with linear ZFE requires higher computation since it needs matrix inversion.

III. Outage Probability Analysis with Relay Selection

In order to derive a RS method which can achieve the optimal DMT, we use the MFB where a single symbol $x[0]$ is sent by the transmitter with energy $E[|x[0]|^2] = P/W$. The transmission is divided into two phases. In the first phase, the source broadcasts the message to the destination and the

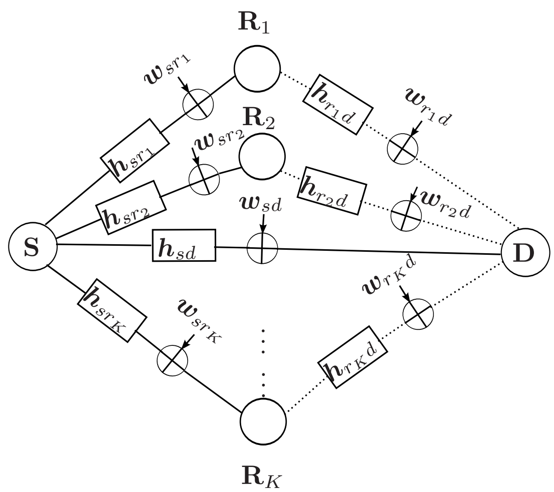

Fig. 1. System model.

relays, and the received signal at the destination and at the ith relay is given by

$$\begin{aligned}\boldsymbol{y}_{sd} &= \boldsymbol{h}_{sd}x[0] + \boldsymbol{w}_{sd}, \\ \boldsymbol{y}_{r_i} &= \boldsymbol{h}_{sr_i}x[0] + \boldsymbol{w}_{sr_i}.\end{aligned}$$

In the second phase, we assume only one selected relay forwards the scaled version of what it receives in the first phase to the destination. Assuming the selected relay is of subscript r_i, we have the received signal at the destination

$$\boldsymbol{y}_{r_id} = \beta_i \boldsymbol{H}_{r_id}(\boldsymbol{h}_{sr_i}x_0 + \boldsymbol{w}_{sr_i}) + \boldsymbol{w}_{r_id}$$

where the amplification factor $\beta_i = \sqrt{\frac{L_{sr_i}}{\|\boldsymbol{h}_{\boldsymbol{sr_i}}\|^2 + L_{sr_i}\rho^{-1}}}$, $\boldsymbol{H}_{r_id} \in \mathbb{C}^{(L_{sr_i}+L_{r_id}-1)\times L_{sr_i}}$ is the Tœplitz channel matrix corresponding to $\boldsymbol{h}_{r_id}$, e.g. $[\boldsymbol{H}_{r_id}]_{m,n} = \boldsymbol{h}_{r_id}[m-n]$.

We next aim to find the mutual information $I(x_0; \boldsymbol{y}_{sd}, \boldsymbol{y}_{r_id})$. This is the prerequisite step for finding the relay selection method which can maximize the mutual information between the sent symbol and the received signals over all possible ways of selecting a single relay. In the following, we first whiten the colored noise in the received signal at the second phase to simplify the calculation. With the assumption that the ith relay is selected, the noise covariance in $\boldsymbol{y}_{r_id}$ can be found as $N_0\boldsymbol{K}_i$ with

$$\boldsymbol{K}_i = \beta_i^2 \boldsymbol{H}_{r_id}\boldsymbol{H}_{r_id}^H + \boldsymbol{I} = \boldsymbol{L}_i^H \boldsymbol{L}_i$$

as $\boldsymbol{K}_i$ is symmetric positive definite and can be LU decomposed. With the similar sufficient statistic argument as for [7, 5.26], we can write the mutual information as

$$\begin{aligned}I(x_0; \boldsymbol{y}_{sd}, \boldsymbol{y}_{r_id}) &= I(x_0; \boldsymbol{y}_{sd}, \boldsymbol{L}_i^{-1}\boldsymbol{y}_{r_id}) \\ &= \frac{1}{2}\log(1 + \rho\|\boldsymbol{h}_{sd}\|^2 + \mathrm{SNR}_{\mathrm{MFB},r_i})\end{aligned}$$

where the SNR in the transformed signal is defined as

$$\mathrm{SNR}_{\mathrm{MFB},r_i} = \rho\beta_i^2 \boldsymbol{h}_{sr_i}^H \boldsymbol{H}_{r_id}^H \boldsymbol{K}_i^{-1} \boldsymbol{H}_{r_id}\boldsymbol{h}_{sr_i}.$$

As we want to maximize $I(x_0; \boldsymbol{y}_{sd}, \boldsymbol{y}_{r_id})$, we need to select the relay $\mathbf{R}_*$ such that

$$\mathrm{SNR}_{\mathrm{MFB},r_*} = \arg\max_{i=1,\cdots,K} \mathrm{SNR}_{\mathrm{MFB},r_i}. \tag{2}$$

To help analyze the performance of the relay selection method (3), we define the singular value decomposition (SVD) of $\boldsymbol{H}_{r_id}$ as $\boldsymbol{U\Sigma V}^H$, and λ_k is the square of kth singular value of $\boldsymbol{H}_{r_id}$. We can write

$$\begin{aligned}\mathrm{SNR}_{\mathrm{MFB},r_i} &= \rho\beta_i^2 \boldsymbol{h}_{sr_i}^H \boldsymbol{H}_{r_id}^H \boldsymbol{K}_i^{-1} \boldsymbol{H}_{r_id}\boldsymbol{h}_{sr_i} \\ &= \rho \sum_{k=0}^{L_{h_{sr_i}}-1} |\boldsymbol{h}'_k|^2 \frac{\beta_i^2\lambda_k}{\beta_i^2\lambda_k + 1}\end{aligned}$$

where $\boldsymbol{h}'_{sr_i} = \boldsymbol{V}\boldsymbol{h}_{sr_i}$. Equivalently, it corresponds to select the relay with the index

$$m = \arg\max_{i=1,\cdots,K} \mathrm{SNR}_{\mathrm{MFB},r_i}. \tag{3}$$

By definition, the outage probability can be expressed as

$$\begin{aligned}P_{out} &= Pr(I(x_0; \boldsymbol{y}_{sd}, \boldsymbol{y}_{r_*d}) < R) \\ &= Pr(\frac{1}{2}\log(1 + \rho\|\boldsymbol{h}_{sd}\|^2 + \mathrm{SNR}_{\mathrm{MFB},r_*}) < R) \qquad (4)\\ &\dot{\leq} Pr((\max(\rho\|\boldsymbol{h}_{sd}\|^2, \mathrm{SNR}_{\mathrm{MFB},r_*}) < 2^{2R}-1) \\ &\doteq Pr(\|\boldsymbol{h}_{sd}\|^2 < \rho^{2r-1}) \prod_{i=1,\cdots,K} Pr(\mathrm{SNR}_{\mathrm{MFB},r_i} < 2^{2R}-1) \\ &= \rho^{(2r-1)(L_{sd}+\sum_{i=1}^K \min(L_{sr_i}, L_{r_id}))} \qquad (5)\end{aligned}$$

where $r = \lim_{\rho\to\infty}\frac{\log R}{\log\rho}$, and (5) follows because

$$\begin{aligned}&\Pr(\mathrm{SNR}_{\mathrm{MFB},r_i} < 2^{2^R}-1) \\ =\; &\Pr(\sum_{k=0}^{L_{h_{sr_i}}-1} |\boldsymbol{h}'_k|^2 \frac{\beta_i^2\lambda_k}{\beta_i^2\lambda_k+1} < \rho^{2r-1}) \\ \doteq\; &\rho^{-\min(L_{sr_i}, L_{r_id})}, \qquad (6)\end{aligned}$$

which is proven in the appendix of [8]. The outage analysis shows that the RS as shown in (3) can achieve the optimal DMT.

IV. Relay Selection with Practical Transceiver Design

While in Section III the outage analysis shows that the proposed RS method can achieve both spatial diversity and frequency diversity; as outage probability is the lower bound on error probability, we might expect such limit can be achieved with MLSE. We note that MLSE has a complexity which increases exponentially with the number of fading taps in the channel. If low-complexity processing is preferred at the destination, an alternative RS method and transceiver structures are needed to achieve full diversity. We next proceed by first describing such a transmission scheme, then analyzing the BER when a specific relay is always selected. Based on analysis of always preselecting a specific relay, we propose a RS method to asymptotically achieves the optimal DMT.

A. Transmission Scheme

The source and relays use the insertion of guard time between blocks of symbols, which can eliminate the possibility of inter-block interference, but not the possibility of inter-symbol interference. To specify the length of guard interval, we define

$$L_{\max} \geq \max_{i \in 1,\ldots,K} \{L_{sr_i}, L_{r_i d}, L_{sd}\}$$

and it is essentially an upper bound on the length of all channels in the system. The transmission of a complete message is divided into two phases:

1) In phase one, after the source broadcasts $\boldsymbol{x}$, a block of N QAM-symbols to the destination and the relays, a guard interval of length $L_{\max} - 1$ zeros follows.
2) In phase two, the source is silent. The selected relay forwards its received signal to the destination under the constraint on transmit power, and then a guard interval of length $L_{\max} - 1$ follows.

In the proposed scheme, QAM-symbols transmitted by the source are drawn from a constellation of $M = \rho^{r'}$ points. We assume that M is a perfect square so that the QAM constellation is well formed [9]. To find the multiplexing gain r' and also make a fair comparison with the cut-set bound on DMT, we want to make sure that

$$\frac{N}{2N + 3L_{\max} - 3} \log M = \frac{1}{2} r \log \rho,$$

thus we found that

$$r' = 1 - \frac{\frac{3}{2}L_{\max} - 1}{N + \frac{3}{2}L_{\max} - \frac{3}{2}}. \tag{7}$$

Due to the insertion of guard time between alternating phases of source/relay transmission, we see from (7) that the system incurs a rate penalty that can be made arbitrarily small by increasing the block length N.

B. BER Analysis with Relay Selection and Linear Zero-Forcing Equalization

In this subsection, we first assume a single relay with index c is always selected without using any channel state information. In the first phase, the received signal at the destination is

$$\boldsymbol{y}_{sd} = \boldsymbol{H}_{sd}\boldsymbol{x} + \boldsymbol{w}_{sd}$$

where $\boldsymbol{H}_{sd} \in \mathbb{C}^{(N+L_{sd}-1)\times N}$ is the Tœplitz channel convolution matrix corresponding to $\boldsymbol{h}_{sd}$ and $\boldsymbol{w}_{sd}$ is the noise at the destination. And the received signal at the relay is

$$\boldsymbol{y}_{r_c} = \boldsymbol{H}_{sr_c}\boldsymbol{x} + \boldsymbol{w}_{r_c}$$

where $\boldsymbol{H}_{sr_c} \in \mathbb{C}^{(N+L_{sr_c}-1)\times N}$ is the Tœplitz channel convolution matrix corresponding to $\boldsymbol{h}_{sr_c}$, $\boldsymbol{w}_{r_c}$ is the noise at the selected relay. The received signal in the second phase from the selected relay to the destination is

$$\begin{aligned}\boldsymbol{y}_{rd} &= \beta_c \boldsymbol{H}_{r_c d}\boldsymbol{y}_{r_c} + \boldsymbol{w}_{r_c d} \\ &= \beta_c \boldsymbol{H}_{r_c d}\boldsymbol{H}_{sr_c}\boldsymbol{x} + \beta_c \boldsymbol{H}_{r_c d}\boldsymbol{w}_{r_c} + \boldsymbol{w}_{r_c d}\end{aligned}$$

where the amplification factor

$$\beta_c = \sqrt{\frac{N + L_{sr_c} - 1}{N\|\boldsymbol{h}_{sr_c}\|^2 + (N + L_{sr_c} - 1)\rho^{-1}}},$$

$\boldsymbol{H}_{r_c d} \in \mathbb{C}^{(L_{sr_c}+L_{r_c d}+N-2)\times(L_{sr_c}+N-1)}$ is the Tœplitz channel convolution matrix corresponding to $\boldsymbol{h}_{r_c d}$, and $\boldsymbol{w}_{r_c d}$ is the noise at the destination when the selected relay transmits. The noise covariance matrix is $\boldsymbol{R}_{ww}N_0$ where $\boldsymbol{R}_{ww} = \beta_c^2 \boldsymbol{H}_{r_c d}\boldsymbol{H}_{r_c d}^H + \boldsymbol{I}$ and can be decomposed as $\boldsymbol{R}_{ww} = \boldsymbol{L}^H\boldsymbol{L}$ (i.e. Cholesky decomposition). After applying the whitening filter $\boldsymbol{L}^{-H}$ to $\boldsymbol{y}_{rd}$, we have

$$\begin{aligned}\tilde{\boldsymbol{y}}_{rd} &= \boldsymbol{L}^{-H}\boldsymbol{y}_{rd} \\ &= \beta_c \boldsymbol{L}^{-H}\boldsymbol{H}_{r_c d}\boldsymbol{H}_{sr_c}\boldsymbol{x} + \tilde{\boldsymbol{w}}\end{aligned}$$

where $\tilde{\boldsymbol{w}} = \boldsymbol{L}^{-H}(\beta_c \boldsymbol{H}_{r_c d}\boldsymbol{w}_{r_c} + \boldsymbol{w}_{d_c})$ is white. Denote $\boldsymbol{G}_c \triangleq \beta_c \boldsymbol{L}^{-H}\boldsymbol{H}_{r_c d}\boldsymbol{H}_{sr_c}$ and define

$$\boldsymbol{H}_{\text{eff},c} = \begin{bmatrix}\boldsymbol{H}_{sd} \\ \boldsymbol{G}_c\end{bmatrix}, \quad \boldsymbol{w}_{\text{eff}} = \begin{bmatrix}\boldsymbol{w}_{sd} \\ \tilde{\boldsymbol{w}}\end{bmatrix}.$$

The signal to be equalized at the destination is then given by

$$\boldsymbol{y} = \boldsymbol{H}_{\text{eff},c}\boldsymbol{x} + \boldsymbol{w}_{\text{eff}}. \tag{8}$$

We note that this model includes the guard intervals inserted between the two transmission phases as can be seen by the dimensions of $\boldsymbol{H}_{sr_c}$, $\boldsymbol{H}_{sd}$, and $\boldsymbol{H}_{r_c d}$. We also note

$$\boldsymbol{H}_{\text{eff},c}^H\boldsymbol{H}_{\text{eff},c} = \boldsymbol{H}_{sd}^H\boldsymbol{H}_{sd} + \boldsymbol{G}_c^H\boldsymbol{G}_c. \tag{9}$$

Denote the minimum eigenvalue of $\boldsymbol{G}_c^H\boldsymbol{G}_c$ as $\lambda_{gc,\min}$, the minimum eigenvalue of $\boldsymbol{H}_{sd}^H\boldsymbol{H}_{sd}$ as $\lambda_{sd,\min}$, and the minimum eigenvalue of $\boldsymbol{H}_{\text{eff},c}^H\boldsymbol{H}_{\text{eff},c}$ as $\lambda_{\text{eff},\min}$. From (9) and the fact that these three matrices are Hermitian, Weyl's Inequality [10, Theorem 4.3.1] gives

$$\lambda_{\text{eff},\min} \geq \lambda_{sd,\min} + \lambda_{gc,\min}. \tag{10}$$

Since $\|\boldsymbol{h}_{sd}\|^2 \neq 0$ with probability 1 and due to [9, Lemma IV.1], $\lambda_{sd,\min} \geq \|\boldsymbol{h}_{sd}\|^2\bar{\lambda}_{sd} > 0$ where

$$\begin{aligned}\bar{\lambda}_{sd} &= \inf_{\boldsymbol{h}_{sd}\in\mathbb{C}^{L_{sd}}} \lambda_{sd,\min}(\bar{\boldsymbol{H}}_{sd}^H\bar{\boldsymbol{H}}_{sd}) > 0, \\ \bar{\boldsymbol{H}}_{sd} &\triangleq \tfrac{\boldsymbol{H}_{sd}}{\|\boldsymbol{h}_{sd}\|}.\end{aligned}$$

Thus we have $\lambda_{\text{eff},\min} > 0$ and $\boldsymbol{H}_{\text{eff},c}^H\boldsymbol{H}_{\text{eff},c}$ is invertible. The receiver at the destination processes the received signal with a ZF equalizer

$$\boldsymbol{F} = (\boldsymbol{H}_{\text{eff}}^H\boldsymbol{H}_{\text{eff},c})^{-1}\boldsymbol{H}_{\text{eff}}^H.$$

The filtered estimate of $\boldsymbol{x}$ at the receiver is then

$$\begin{aligned}\tilde{\boldsymbol{y}} &= \boldsymbol{F}\boldsymbol{y} \\ &= \boldsymbol{x} + (\boldsymbol{H}_{\text{eff},c}^H\boldsymbol{H}_{\text{eff},c})^{-1}\boldsymbol{H}_{\text{eff},c}^H\boldsymbol{w}_{\text{eff}}.\end{aligned}$$

The filtered noise $\boldsymbol{z} = (\boldsymbol{H}_{\text{eff},c}^H\boldsymbol{H}_{\text{eff},c})^{-1}\boldsymbol{H}_{\text{eff},c}^H\boldsymbol{w}_{\text{eff}}$ has a total variance

$$\begin{aligned}E\left[\|\boldsymbol{z}\|^2\right] &= E[\boldsymbol{z}^H\boldsymbol{z}] \\ &= \text{tr}[(\boldsymbol{H}_{\text{eff},c}^H\boldsymbol{H}_{\text{eff},c})^{-1}]N_0.\end{aligned}$$

Assuming each symbol in the block is estimated separately, we find that the effective SNR for decoding the kth symbol is

$$\begin{aligned}
\mathrm{SNR}_{\mathrm{eff}}(k) &= \frac{P}{WE[|z_k|^2]} \geq \frac{P}{WE[\|\boldsymbol{z}\|^2]} \\
&= \frac{\rho}{\mathrm{tr}[(\boldsymbol{H}_{\mathrm{eff},c}^H \boldsymbol{H}_{\mathrm{eff},c})^{-1}]} \\
&= \frac{\rho}{\sum_{k=0}^{N-1} \lambda_{\mathrm{eff},k}^{-1}} \geq N\rho\lambda_{\mathrm{eff,min}} \\
&\geq \frac{1}{N}\rho(\lambda_{sd,\min} + \lambda_{gc,\min})
\end{aligned}$$

where $\lambda_{\mathrm{eff},k}$ is the kth eigenvalue for $\boldsymbol{H}_{\mathrm{eff},c}^H \boldsymbol{H}_{\mathrm{eff},c}$.

The error probability at the destination [9, Lemma VII.6] is

$$\begin{aligned}
P_e &\doteq \Pr[\mathrm{SNR}_{\mathrm{eff}}(k) < \rho^{2r'}] \\
&\leq \Pr[\frac{1}{N}\rho(\lambda_{sd,\min} + \lambda_{gc,\min}) < \rho^{2r'}] \\
&\leq \rho^{-(L_{sd}+\min\{L_{sr_c}, L_{r_c d}\})(1-2r')} \qquad (11)
\end{aligned}$$

where the detailed derivation of (11) can be found in [8]. We see that pre-selection without any knowledge of channel state does not exploit any spatial diversity.

We next aim to find a single RS method which can exploit the distributed space diversity and hence asymptotically achieve the optimal DMT with linear ZFE. Hence, we focus on (**??**) and select the relay which can give the largest lower bound on the effective SNR as shown in (11), i.e., the relay with the index

$$m = \arg\max_i \frac{\rho}{\mathrm{tr}[(\boldsymbol{H}_{\mathrm{eff},i}^H \boldsymbol{H}_{\mathrm{eff},i})^{-1}]}. \qquad (12)$$

It is equivalent to select the relay with the largest average decision-point (DP) SNR and is represented by "Max DP SNR". With this RS method, we can calculate the error probability at the destination [9, Lemma VII.6] as

$$\begin{aligned}
P_e &\doteq \Pr[\mathrm{SNR}_{\mathrm{eff}}(k) < \rho^{2r'}] \\
&\leq \Pr[\max_{i=1,\cdots,K} \frac{\rho}{\mathrm{tr}[(\boldsymbol{H}_{\mathrm{eff},i}^H \boldsymbol{H}_{\mathrm{eff},i})^{-1}]} < \rho^{2r'}] \\
&\leq \Pr[\frac{\rho}{N}(\lambda_{sd,\min} + \max_{i=1,\cdots,K} \lambda_{gi,\min}) < \rho^{2r'}] \qquad (13) \\
&= \rho^{-(L_{sd}+\sum_{i=1}^K \min\{L_{sr_i}, L_{r_i d}\})(1-2r')} \qquad (14)
\end{aligned}$$

where the "max" in (13) suggests the spatial diversity can be exploited and again the detailed derivation of (14) can be found in [8]. Combining the result in (14) and the upper bound on DMT in (1), we have the following result on DMT of this multiple amplify-and-forward relay system

$$\begin{aligned}
&(L_{sd} + \sum_{i=1}^K \min(L_{sr_i}, L_{r_i d}))(1-2r') \dot{\leq} d(r) \\
&\dot{\leq} (L_{sd} + \sum_{i=1}^K \min(L_{sr_i}, L_{r_i d}))(1-2r).
\end{aligned}$$

As we can see in (7), as the block length N goes into infinity, r' is asymptotically equal to r. Thus, this RS method asymptotically achieves the upper bound on DMT with ZFE at the destination.

The RS method in (12) requires full CSI and K matrix inversion; consequently, the computation complexity is relatively high. The implementation of RS can be completed in a distributed fashion by relays or a centralized fashion at the destination. However, in a distributed fashion, it should be noted that the CSI of the source-to-destination channel should be transmitted to each relay. It is interesting to note that in flat fading, this RS method and the method in (3) coincide.

V. Numerical Results

This section presents numerical examples for the proposed RS methods developed in Section III and Sections IV. In evaluating performance over finite SNRs, the diversity measured as the negative slope of each outage curve often does not coincide exactly with the predicted maximal diversity [11], [12]; the predicted diversity assumes that the SNR grows arbitrarily large to permit the analysis to be mathematically tractable.

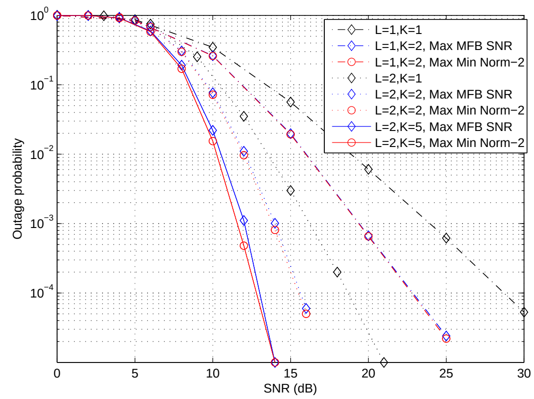

Fig. 2. Simulated outage probability for the two RS methods, $R = 2$ bits/s/Hz.

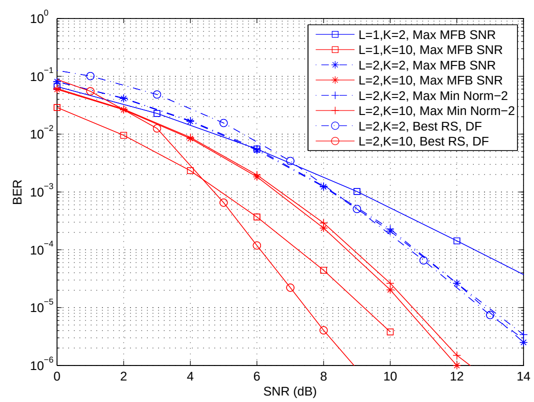

Fig. 3. Simulated BER for i.i.d. fading channels with MLSE and QPSK.

Fig. 2 shows the outage probability where the rate $R = 2$ bits/s/Hz and each fading tap of $\boldsymbol{h}_{jk}$ is i.i.d. Rayleigh fading with variance 1 where the subscript jk can be sd, sr_i or $r_i d$. In the figure, "Max MFB SNR" indicates the RS method in (3). We also simulate another RS method, namely "Max Min Norm-2", which selects the relay with the index

$$m = \arg\max_{i=1,\cdots,K} \min(\|\boldsymbol{h}_{sr_i}\|^2, \|\boldsymbol{h}_{r_i d}\|^2) \qquad (15)$$

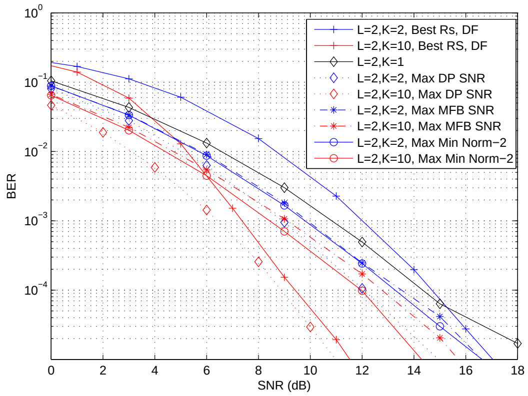

Fig. 4. Simulated BER for i.i.d. fading channels with linear ZFE and QPSK.

for purposes of comparison. As we can see, both the "Max MFB SNR" method and "Max Min Norm-2" method result in the same outage probability. As the number of relays increases, the negative slopes of the outage curves also increase. Therefore both RS methods can exploit the spatial diversity. When the number of relay remains the same, each outage curve of $L = 2$ has a larger negative slope than the corresponding outage curve of $L = 1$. This shows the frequency diversity can be exploited. Hence both the RS methods can provide a full-diversity channel.

We further verify the BER performance of the transceiver designs with different relaying strategies. In the simulation, we use a block length $N = 32$ and Gray-mapped QPSK symbols. We consider frequency selective channels with uniform power delay profile, i.e. each tap of each channel is i.i.d. fading with variance $1/L$ where L is the channel length. We first consider the performance of (MLSE) with the "Max MFB SNR" and "Max Min Norm-2" and we employ a two-tap whitening filter before MSLE to whiten the noise at destination which is colored through the AF relay. The "Best RS, DF" represents the best RS method we proposed in [6] with DF protocol. As shown in Fig. 3, both RS methods have better performance when more relays are present in the system. However, in the high SNR region, compared to the case with DF relay networks, the power gain with increased number of relays is much less. We also note that in the low SNR region, RS with the AF protocol has better performance than RS with DF protocol.

We plot the BER performance of the transceiver design based on linear ZFE in Fig. 4. As shown in the figure, it provides improved BER performance with increased number of relays. It also reaps almost the same diversity order as the best RS method with DF protocol. [6]. Compared to the best RS with DF protocol, the "Max DP SNR" relay method with AF protocol has better BER performance with the cost of much more computation in performing RS. While the RS methods represented by "Max MFB SNR" and "Max Min Norm-2" can exploit spatial diversity with MLSE at the destination, the RS methods represented by "Max MFB SNR" and "Max Min Norm-2" cannot exploit spatial diversity with linear ZFE. This results from the small correlation between the selection criterion to the actual decision-point SNR which dictates the BER performance. This is also different from RS with DF protocol where the best RS method in [6] can achieve full diversity, no matter what equalization is used.

VI. Conclusion

In this paper, we considered relay selection for an orthogonal AF system where i.i.d. FS fading is present. We analyzed the outage performance based on the MFB and developed a RS method that achieves the optimal DMT. The RS method selects the relay with the largest decision-point SNR at the destination when only one symbol is sent. This RS scheme is useful in exploiting spatial diversity when MLSE is used at the destination, and this result is corroborated by simulation. We also developed a RS method when the destination node uses linear ZFE. It selects the relay with the largest average DP SNR. While this method is proven to asymptotically achieve the optimal DMT, the computation complexity in performing relay selection is relatively high since this method requires matrix inversions where the number of required matrix inversions is up to the number of relays in the system, and the matrix dimension is up to the block length. This informs us that for full-diversity AF RS schemes in frequency selective channels, the RS method is closely connected to the equalization method at the destination.

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