SUBSPACE-BASED BLIND CHANNEL IDENTIFICATION FOR ORTHOGONAL MODULATION

A.G. Klein and P. Duhamel

Supélec/LSS
3, rue Joliot-Curie
91192 Gif-sur-Yvette, FRANCE

ABSTRACT

We consider blind channel estimation for orthogonal modulation and its variants — which include pulse position modulation (PPM) and frequency shift keying (FSK). While equalization of this modulation format has been given some attention by the research community, little attention has been paid to techniques for (blind) channel estimation in systems employing orthogonal modulation. We extend classical subspace-based blind techniques in a way that is suitable for use with orthogonal modulation. Unlike classical subspace-based blind channel estimators, however, our scheme does not require oversampling and/or multiple sensors. After introducing the system model, we present the proposed method of channel estimation, and include conditions under which the scheme is valid. We conclude with several simulations.

1. INTRODUCTION

Orthogonal modulation is a modulation scheme that has been studied for many applications, and its many variants include frequency shift keying (FSK) and pulse position modulation (PPM). Traditional uses of orthogonal modulation have been in situations with little or no ISI, and thus there has been little motivation to explore channel identification of such signals to the extent that channel identification has been explored for linearly modulated signals. Orthogonal modulation is a power efficient scheme, but is bandwidth inefficient, and thus has attracted attention for use in ultra wideband (UWB) communication systems where ISI is an issue.

Several schemes have been proposed for equalization of orthogonally modulated signals, for example [1][2][3]. However, all of these schemes rely on previous knowledge of the channel taps, or they assume that training data is available. In situations where training data is unavailable or insufficient, blind channel identification schemes are required. In this paper, we propose a technique for subspace-based blind channel identification for use with orthogonal modulations. While we believe this to be the first work addressing blind channel identification for the general class of orthogonal modulations, we point out several related works. First, we note that our technique is an extension of that proposed in [4]. In the context of orthogonal modulation, however, our technique has the benefit of not requiring oversampling or multiple sensors as was necessary in [4]. Also related is [5], wherein the authors propose subspace-based detection for multiuser ultra-wideband (UWB) environments. While PPM is considered in [5], the conditions for successful channel identification are not provided.

We begin by describing orthogonal modulation and the corresponding system model. Then we present a channel identification method, which, it turns out, is suitable for use with a general class of modulation formats that exhibit particular cyclostationary source statistics. We include the identifiability conditions for the method, and specialize the method for use with orthogonal modulations. Next, we present simulations that demonstrate algorithm performance, and we conclude the paper. In this paper, we use $^\top$ to denote matrix transpose, ^{H} to denote Hermitian transpose, $\otimes$ to denote matrix direct (Kronecker) product, $\boldsymbol{I}_m$ to denote the $m \times m$ identity matrix, $\mathbf{1}_{m\times n}$ to denote the $m \times n$ matrix of all ones, $\mathbf{0}_{m\times n}$ to denote the $m \times n$ matrix of all zeros, and $\boldsymbol{e}_i$ to denote the unit canonical vector with a 1 in the ith element.

2. SYSTEM MODEL

M-ary orthogonal modulation is accomplished by transmitting one of M orthogonal waveforms serially through the channel. Each waveform is assumed to consist of M chips, so orthogonal modulation can be thought of as a block coding scheme where information is conveyed by transmitting one of M codes. Let $\boldsymbol{S} \in \mathbb{C}^{M\times M}$ be the matrix whose columns are the M waveforms. We assume the waveforms are mutually orthogonal and have unit energy, so that $\boldsymbol{S}^H\boldsymbol{S} = \boldsymbol{I}_M$. Thus, for example, the choice $\boldsymbol{S} = \boldsymbol{I}_M$ corresponds to PPM, while choosing $\boldsymbol{S}$ to be the DFT matrix where $[\boldsymbol{S}]_{i,j} = e^{j2\pi ij/M}$ corresponds to FSK. In this paper, we consider a coherent sampled model where each chip is sampled once.

A.G. Klein was supported by a Chateaubriand Scientific Fellowship.
Contact email: klein@lss.supelec.fr

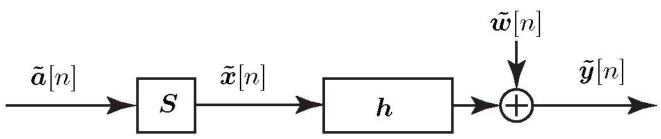

Fig. 1. System Model

The system model is shown in Fig. 1. To represent orthogonal modulation, we assume that $\log_2 M$ bits are mapped to one of M selection vectors $\boldsymbol{a}[n]$ which are simply unit vectors that serve to select the desired waveform, so that $\boldsymbol{a}[n] \in \{\boldsymbol{e}_0, \boldsymbol{e}_1, \ldots, \boldsymbol{e}_{M-1}\}$. Next, the matrix $\boldsymbol{S}$ maps the selection vectors to one of the orthogonal waveforms, so that the symbol waveform transmitted at time n is given by

$$\boldsymbol{x}[n] = \boldsymbol{S}\boldsymbol{a}[n] \tag{1}$$

where $\boldsymbol{x}[n] \in \mathbb{C}^M$. The M-ary orthogonal symbols are assumed to be i.i.d. and equiprobable, and we note that the resulting chip-rate process is cyclostationary with period M. The chips are transmitted serially through a causal linear time-invariant channel having finite impulse response given by $\boldsymbol{h} = [h[0], h[1], \ldots, h[N_h - 1]]^\top$. The channel also contributes zero-mean additive white Gaussian noise (AWGN) of variance σ_w^2.

Since the chips are transmitted serially through the channel, we use the stacked vector

$$\tilde{\boldsymbol{x}}[n] = \begin{bmatrix} \boldsymbol{x}[n] \\ \boldsymbol{x}[n-1] \\ \vdots \end{bmatrix} \tag{2}$$

so that N samples of the the received signal can be written

$$\tilde{\boldsymbol{y}}[n] = \boldsymbol{\mathcal{H}}_N \tilde{\boldsymbol{x}}[n] + \tilde{\boldsymbol{w}}[n] \tag{3}$$

where $\tilde{\boldsymbol{y}}[n] \in \mathbb{R}^N$, $\tilde{\boldsymbol{x}}[n] \in \mathbb{R}^{N+N_h-1}$, the Tœplitz channel matrix $\boldsymbol{\mathcal{H}}_N \in \mathbb{R}^{N \times N+N_h-1}$ is defined as $[\boldsymbol{\mathcal{H}}_N]_{i,j} = h[j-i]$, and $\tilde{\boldsymbol{w}}[n] \in \mathbb{R}^N$ is the AWGN. We note that $\boldsymbol{\mathcal{H}}_N$ is a wide matrix that is full row rank, but is certainly not full column rank. Just as $\tilde{\boldsymbol{x}}[n]$ contains the stacked transmitted symbol vectors, we can encapsulate the corresponding selection vectors into $\tilde{\boldsymbol{a}}[n] = [\boldsymbol{a}^\top[n],\ \boldsymbol{a}^\top[n-1],\ \ldots]^\top$, which enables us to write $\tilde{\boldsymbol{x}}[n] = (\boldsymbol{I} \otimes \boldsymbol{S})\tilde{\boldsymbol{a}}[n]$, and

$$\tilde{\boldsymbol{y}}[n] = \boldsymbol{\mathcal{H}}_N(\boldsymbol{I} \otimes \boldsymbol{S})\tilde{\boldsymbol{a}}[n] + \tilde{\boldsymbol{w}}[n]. \tag{4}$$

A typical receiver would then perform some method of ISI compensation before making a decision on which symbol was transmitted. As the receiver may have no *a priori* information about the channel impulse response coefficients, we now consider a blind scheme for estimation of these parameters.

3. SUBSPACE-BASED IDENTIFICATION

3.1. Preliminaries

Having established the system model for transmission of orthogonal modulation, we now shift our attention to a blind method for estimating the N_h channel coefficients $\boldsymbol{h}$ from the received signal $\tilde{\boldsymbol{y}}[n]$. Our method is an extension of [4], which relies on subspace concepts and second-order statistics of the received signal. In classical methods for subspace-based blind channel identification [4][6], two assumptions are made:

A1. Oversampling or multiple sensors are employed (and necessary).

A2. The autocovariance matrix of the transmitted source data (i.e. $E[\tilde{\boldsymbol{x}}[n]\tilde{\boldsymbol{x}}^H[n]]$) is full-rank.

However, *neither* of these assumptions are made here; in fact, we show that violation of A2 is precisely what eliminates the need for oversampling in A1. Hence, our method is effectively an extension of [4] to operate on source data with rank-deficient autocovariance matrices (which is the case for orthogonal modulation as we will show) without the need for oversampling.

Before specializing the method to M-ary orthogonal modulation, however, we first consider a more general situation. For now, we only assume that $\tilde{\boldsymbol{x}}[n]$ is a period M cyclostationary source sequence arising from i.i.d. symbols $\boldsymbol{x}[n] \in \mathbb{R}^M$ constructed as in (2). We denote the mean $\boldsymbol{\mu}_x \triangleq E[\boldsymbol{x}[n]]$ and autocovariance matrix of a single symbol $\boldsymbol{\gamma}_x \triangleq E[(\boldsymbol{x}[n] - \boldsymbol{\mu}_x)(\boldsymbol{x}[n] - \boldsymbol{\mu}_x)^H]$, and we assume $r \triangleq \mathrm{rank}(\boldsymbol{\gamma}_x) < M$ so that A2 is violated. Since $\boldsymbol{\gamma}_x$ is an autocovariance matrix, it is Hermitian symmetric and semi-positive definite, and therefore admits a factorization

$$\boldsymbol{\Gamma}_x\boldsymbol{\Gamma}_x^H = \boldsymbol{\gamma}_x \tag{5}$$

where $\boldsymbol{\Gamma}_x \in \mathbb{C}^{M \times r}$ is full column rank. Using these definitions together with (2) gives the first- and second-order statistics of the transmitted vector $\tilde{\boldsymbol{x}}[n]$ as

$$\begin{aligned} \boldsymbol{\mu}_{\tilde{x}} &\triangleq E[\tilde{\boldsymbol{x}}[n]] \\ &= \mathbf{1}_{N_x \times 1} \otimes \boldsymbol{\mu}_x \end{aligned} \tag{6}$$

$$\begin{aligned} \boldsymbol{\gamma}_{\tilde{x}} &\triangleq E[(\tilde{\boldsymbol{x}}[n] - \boldsymbol{\mu}_{\tilde{x}})(\tilde{\boldsymbol{x}}[n] - \boldsymbol{\mu}_{\tilde{x}})^H] \\ &= \boldsymbol{I}_{N_x} \otimes \boldsymbol{\gamma}_x \end{aligned} \tag{7}$$

where $N_x \triangleq \frac{N+N_h-1}{M}$, and we have used the fact that symbols are i.i.d. in arriving at (7). For simplicity, we assume the observation window length N is chosen so that $N + N_h - 1$ is a multiple of M.

Similar to [4][6], the blind identification method we propose is based on the autocovariance matrix of the received signal $\tilde{\boldsymbol{y}}[n]$. While the receiver does not have exact knowledge of the autocovariance matrix, a sample estimate can be used. For now, however, we assume the receiver has exact knowledge of the autocovariance matrix. From (3) and (5)-(6), we have

$$\begin{aligned}\boldsymbol{\mu}_{\tilde{y}} &\triangleq E[\tilde{\boldsymbol{y}}[n]] \\ &= \boldsymbol{\mathcal{H}}_N\left(\mathbf{1}_{N_x\times 1}\otimes\boldsymbol{\mu}_x\right) \\ \boldsymbol{\gamma}_{\tilde{y}} &\triangleq E[(\tilde{\boldsymbol{y}}[n]-\boldsymbol{\mu}_{\tilde{y}})(\tilde{\boldsymbol{y}}[n]-\boldsymbol{\mu}_{\tilde{y}})^H] \\ &= \boldsymbol{\mathcal{H}}_N\boldsymbol{\gamma}_{\tilde{x}}\boldsymbol{\mathcal{H}}_N^H+\sigma_w^2\boldsymbol{I}_N \\ &= \boldsymbol{H}_{eff,N}\boldsymbol{H}_{eff,N}^H+\sigma_w^2\boldsymbol{I}_N \qquad (8)\end{aligned}$$

where the source data and noise are assumed to be uncorrelated, and the effective channel matrix is

$$\boldsymbol{H}_{eff,N}\triangleq\boldsymbol{\mathcal{H}}_N(\boldsymbol{I}_{N_x}\otimes\boldsymbol{\Gamma}_x)\in\mathbb{C}^{N\times(N+N_h-1)r/M}. \qquad (9)$$

As in [4][6], we require $\boldsymbol{H}_{eff,N}$ to be tall and full column rank. From (9), we see that if $r=M$ (i.e. if the data autocovariance matrix is full rank), $\boldsymbol{H}_{eff,N}$ cannot be made tall. Thus, it is precisely the requirement $r<M$ that permits a tall channel matrix for sufficient window length N, thereby enabling the use of subspace-based blind identification schemes.

3.2. Subspace decomposition

Let $\boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^H=\boldsymbol{\gamma}_{\tilde{y}}$ be the eigendecomposition of the autocovariance matrix of the received signal (8), with eigenvalues $\lambda_0\geq\lambda_1\geq\cdots\geq\lambda_{rN_x-1}\geq\lambda_{rN_x}=\cdots=\lambda_{N-1}=\sigma_w^2$. From (8) the *signal part* of the covariance matrix has rank rN_x, due to the assumption that $\boldsymbol{H}_{eff,N}$ is full column rank. We can partition $\boldsymbol{U}$ into signal and noise subspaces as $\boldsymbol{U}=[\boldsymbol{F}\ \boldsymbol{G}]$ where

$$\begin{aligned}\boldsymbol{F} &= [\boldsymbol{f}_0,\ldots,\boldsymbol{f}_{rN_x-1}] \\ \boldsymbol{G} &= [\boldsymbol{g}_0,\ldots,\boldsymbol{g}_{N-rN_x-1}]\end{aligned}$$

so that

$$\boldsymbol{\gamma}_{\tilde{y}}=\boldsymbol{F}\begin{bmatrix}\lambda_0 & & \\ & \ddots & \\ & & \lambda_{rN_x-1}\end{bmatrix}\boldsymbol{F}^H+\sigma_w^2\boldsymbol{G}\boldsymbol{G}^H.$$

The signal subspace (i.e. the linear space that is spanned by the columns of $\boldsymbol{F}$) coincides with the space spanned by the columns of $\boldsymbol{H}_{eff,N}$. By orthogonality of columns in $\boldsymbol{F}$ with those in $\boldsymbol{G}$, any vector in the noise subspace is orthogonal to the columns in $\boldsymbol{H}_{eff,N}$, i.e.

$$\boldsymbol{H}_{eff,N}^H\boldsymbol{g}_k=0_{rN_x\times 1} \qquad (10)$$

for all $k\in\{0,\ldots,N-rN_x-1\}$, which is the principle used in the channel identification procedure. Let $\boldsymbol{\mathcal{G}}_k\in\mathbb{C}^{N_h\times N+N_h-1}$ be the Tœplitz filtering matrix constructed from $\boldsymbol{g}_k$ so that $[\boldsymbol{\mathcal{G}}_k]_{i,j}=[\boldsymbol{G}]_{j-i,k}$. Using the definition of $\boldsymbol{H}_{eff,N}$ in (9), we can rewrite (10) as

$$\begin{aligned}(\boldsymbol{I}_{N_x}\otimes\boldsymbol{\Gamma}_x^H)\boldsymbol{\mathcal{H}}_N^H\boldsymbol{g}_k &= 0_{rN_x\times 1} \\ \text{or}\quad (\boldsymbol{I}_{N_x}\otimes\boldsymbol{\Gamma}_x^H)\left(\boldsymbol{\mathcal{G}}_k^H\boldsymbol{h}\right)^* &= 0_{rN_x\times 1} \qquad (11)\end{aligned}$$

where (11) follows from the fact that filtering the vector $\boldsymbol{g}_k$ with the Tœplitz filtering matrix $\boldsymbol{\mathcal{H}}_N$ is equivalent to filtering the vector $\boldsymbol{h}$ with the Tœplitz filtering matrix $\boldsymbol{\mathcal{G}}_k$. As (11) holds for all k, we conjugate both sides and stack the $\boldsymbol{\mathcal{G}}_k$'s to obtain

$$\underbrace{(\boldsymbol{I}\otimes\boldsymbol{\Gamma}_x^T)\begin{bmatrix}\boldsymbol{\mathcal{G}}_0^H \\ \vdots \\ \boldsymbol{\mathcal{G}}_{N-rN_x-1}^H\end{bmatrix}}_{\triangleq\boldsymbol{Q}}\boldsymbol{h} = 0. \qquad (12)$$

3.3. On the identifiability of the solution

To show that (12) can be used to *uniquely* determine the channel coefficients $\boldsymbol{h}$ (with an inherent scalar ambiguity), we prove the following theorem. As in (9), we continue to use the notation $\boldsymbol{H}_{eff,N}$ to denote the $N\times(N+N_h-1)r/M$ effective channel matrix constructed from channel taps $\boldsymbol{h}$. Recall that N is the length of observation window, and therefore the number of rows in $\boldsymbol{H}_{eff,N}$. We use the notation $\boldsymbol{H}_{eff,N-M}$ to denote the (reduced) size $N-M\times(N-M+N_h-1)r/M$ effective channel matrix constructed from the same $\boldsymbol{h}$ but with an observation window of length $N-M$.

Theorem 1. *Assume that $\boldsymbol{H}_{eff,N-M}$ having corresponding channel taps $\boldsymbol{h}$ is full column rank. Let $\boldsymbol{h}'$ be another set of channel taps, with corresponding effective channel matrix $\boldsymbol{H}'_{eff,N}$. Then, the following are equivalent:*

- *S1. The matrix $\boldsymbol{H}'_{eff,N}$ is nonzero and the range of $\boldsymbol{H}'_{eff,N}$ is included in the range of $\boldsymbol{H}_{eff,N}$.*
- *S2. The matrices are related via $\boldsymbol{H}_{eff,N}=\alpha\boldsymbol{H}'_{eff,N}$, where $\alpha\neq 0$ is a a complex scalar factor.*

Proof. We provide a sketch, as the proof is very similar to that in [4, Theorem 3]. That S2 $\Longrightarrow$ S1 is trivial. Thus, we focus on showing S1 $\Longrightarrow$ S2. Following the same procedure in [4, Theorem 3], though with block matrices in place of vectors, it can be shown that, under S1,

$$\boldsymbol{H}_{eff,N}=\boldsymbol{H}'_{eff,N}(\boldsymbol{I}_{N_x}\otimes\boldsymbol{A}) \qquad (13)$$

for some matrix $\boldsymbol{A}\in\mathbb{C}^{r\times r}$. The steps involve a block partitioning of $\boldsymbol{H}_{eff,N}$, and then demonstration that in order for any column of $\boldsymbol{H}_{eff,N}$ to be in the range of $\boldsymbol{H}'_{eff,N}$, (13) must be satisfied.

Alas, the condition (13) is not the result we seek (i.e. S2), and so more work is required. However, $\boldsymbol{A}$ can only assume values such that $\boldsymbol{H}'_{eff,N}(\boldsymbol{I}_{N_x}\otimes\boldsymbol{A})$ is a valid channel matrix (i.e. so that the Tœplitz structure of the underlying $\boldsymbol{\mathcal{H}}'_N$ is preserved). Note that for any choice of $\boldsymbol{A}\in\mathbb{C}^{r\times r}$, $\boldsymbol{\Gamma}_x\boldsymbol{A}$ can be written as $\boldsymbol{B}\boldsymbol{\Gamma}_x$ for some $\boldsymbol{B}\in\mathbb{C}^{M\times M}$, due to the fact that $\boldsymbol{\Gamma}_x$ is full column rank. Thus, for some $\boldsymbol{B}$, we can write

$$\begin{aligned}\boldsymbol{H}'_{eff,N}(\boldsymbol{I}_{N_x}\otimes\boldsymbol{A}) &= \boldsymbol{\mathcal{H}}'_N(\boldsymbol{I}_{N_x}\otimes\boldsymbol{\Gamma}_x)(\boldsymbol{I}_{N_x}\otimes\boldsymbol{A}) \\ &= \boldsymbol{\mathcal{H}}'_N(\boldsymbol{I}_{N_x}\otimes\boldsymbol{B})(\boldsymbol{I}_{N_x}\otimes\boldsymbol{\Gamma}_x)\end{aligned}$$

where the product $\mathcal{H}'_N(\boldsymbol{I}_{N_x} \otimes \boldsymbol{B})$ needs to be a Tœplitz matrix in order to be a valid channel matrix. The product of a rectangular Tœplitz matrix with a square matrix can only itself be Tœplitz if the square matrix is a scaled identity matrix. Hence, $\boldsymbol{B} = \alpha \boldsymbol{I}$, and

$$\boldsymbol{H}_{eff,N} = \alpha \boldsymbol{H}'_{eff,N}.$$

■

Consequently, the noise subspace $\boldsymbol{G}$ uniquely determines the channel coefficients $\boldsymbol{h}$.

3.4. Subspace-based parameter estimation scheme

When the exact autocovariance matrix is available at the receiver, an exact solution to (12) can be found. However, in practice only sample estimates of $\mathcal{G}_k$ are available, and so the nullity of $\boldsymbol{Q}$ in (12) may not be exactly 1. Consequently, we can solve (12) in the least squares sense with a constraint to avoid the trivial solution $\boldsymbol{h} = 0$. As addressed in [4], different constraints provide different solutions; however, we consider only the quadratic constraint $\boldsymbol{h}^H \boldsymbol{h} = 1$. Thus, we can estimate $\boldsymbol{h}$ via the constrained minimization

$$\hat{\boldsymbol{h}} = \arg \min_{||\hat{\boldsymbol{h}}||_2 = 1} \hat{\boldsymbol{h}}^H \hat{\boldsymbol{Q}}^H \hat{\boldsymbol{Q}} \hat{\boldsymbol{h}}$$

which amounts to choosing the eigenvector corresponding to the smallest eigenvalue of $\hat{\boldsymbol{Q}}^H \hat{\boldsymbol{Q}}$. We note that this quadratic constraint requires calculation of an additional eigendecomposition, so other constraints may be preferable from a computational perspective [4].

3.5. Particulars for orthogonal modulation

Up to now, Section 3 has proceeded in a fair amount of generality: the blind channel identification method we have presented is suitable for any period M cyclostationary source sequence with i.i.d. symbols, as long as the rank of the autocovariance of the source vector is not full rank (i.e. $r < M$). Consequently, the scheme is suitable for use with other nonlinear modulation schemes like biorthogonal modulation and simplex modulation, so long as the rank deficient condition is satisfied. Here, however, we only consider use of the scheme for the orthogonal modulation system model described in Section 2.

We now consider the specific form of the autocovariance matrix for a source sequence in orthogonal modulation, and show that it satisfies the rank deficiency requirement. From (1), we have

$$\begin{aligned} \boldsymbol{\mu}_x &= E[\boldsymbol{x}[n]] \\ &= \frac{1}{M} \boldsymbol{S} \boldsymbol{1}_{M \times 1} \\ \gamma_x &= E[\boldsymbol{x}[n] \boldsymbol{x}^H[n]] - \boldsymbol{\mu}_x \boldsymbol{\mu}_x^H \\ &= \frac{1}{M} \boldsymbol{I}_M - \frac{1}{M^2} \boldsymbol{S} \boldsymbol{1}_{M \times M} \boldsymbol{S}^H. \end{aligned} \tag{14}$$

Lemma 1. *The autocovariance matrix γ_x described in (14) has rank $r = M - 1$.*

Proof. First note that $M\gamma_x$ is an idempotent matrix since $(M\gamma_x)^2 = M\gamma_x$, which can be shown using the fact that $\boldsymbol{S}$ is unitary, and $\boldsymbol{1}^2_{M \times M} = M \boldsymbol{1}_{M \times M}$. The rank of an idempotent matrix equals its trace [7], so

$$\begin{aligned} \text{rank}(\gamma_x) &= \text{rank}(M\gamma_x) \\ &= \text{tr}(\boldsymbol{I}_M) - \text{tr}\left(\frac{1}{M} \boldsymbol{1}_{M \times M} \boldsymbol{S}^H \boldsymbol{S}\right) \\ &= M - 1 \end{aligned}$$

■

Hence, the autocovariance matrix is rank deficient as required. The length condition necessary to ensure that the effective channel matrix $\boldsymbol{H}_{eff,N}$ is tall is given by

$$N > (M-1)N_h. \tag{15}$$

Summarizing, then, the estimation procedure is as follows:

1. Generate an estimate of the autocovariance of the received signal, $\hat{\gamma}_{\tilde{y}} \in \mathbb{C}^{N \times N}$ where N satisfies (15).
2. Find the eigendecomposition of $\hat{\gamma}$, and store the eigenvectors corresponding to the $N + N_h - 1 - \lfloor (N + N_h - 1)/M \rfloor$ smallest eigenvalues in $\hat{\boldsymbol{G}}$.
3. Form $\hat{\boldsymbol{Q}}^H \hat{\boldsymbol{Q}}$ where $\boldsymbol{Q}$ was defined in (12).
4. The channel estimate $\boldsymbol{h}$ is then the eigenvector corresponding to the smallest eigenvalue of $\hat{\boldsymbol{Q}}^H \hat{\boldsymbol{Q}}$.

Implicit in the above procedure is that the receiver has knowledge of the channel length N_h, which is a standard assumption of subspace-based blind channel estimators. In practice, this will not be the case. While we assume this information is available to the receiver, the issue of channel order estimation has been addressed in [8], for example.

Also implicit in the above procedure is knowledge of the rank-deficient autocovariance matrix of the source sequence γ_x (and corresponding factorization $\boldsymbol{\Gamma}_x \boldsymbol{\Gamma}_x^H$), which is required for the construction of $\hat{\boldsymbol{Q}}$ in step 3. While this assumption seems very reasonable, we note that it is in contrast to [4] where it was only necessary that γ_x be full rank but otherwise unknown.

4. SIMULATIONS

To investigate the effect of using the *estimated* autocovariance matrix instead of the exact autocovariance matrix, we conducted the following experiment. We chose the symbol waveforms to be binary PPM, so that $\boldsymbol{S} = \boldsymbol{I}_2$. Since PPM signaling only has a real component, we conducted the experiment using only real signals. We used a non-minimum phase length $N_h = 10$ channel with impulse response

$$\boldsymbol{h} = [-0.091, 0.843, -0.372, 0.168, -0.091, -0.015, -0.047, -0.305, -0.102, -0.0270]^\top$$

with AWGN such that the SNR was 10 dB. Note that this channel has three roots very near the unit circle. The length of the temporal window was chosen to be $N = 12$ which satisfies (15). After finding the estimated channel using the proposed blind method, we calculated the mean square error between the exact channel impulse response and estimated response via

$$MSE = (\boldsymbol{h} - \hat{\boldsymbol{h}})^H (\boldsymbol{h} - \hat{\boldsymbol{h}}) \tag{16}$$

and we averaged over 500 Monte Carlo runs. We conducted the simulation for a variety of symbol lengths, and the results are shown by the solid line in Fig. 2. We note that the performance of the channel estimate depends heavily on the amount of AWGN; for example, when we increased the SNR to 20 dB, we found that the algorithm was able to attain an MSE less than -20 dB.

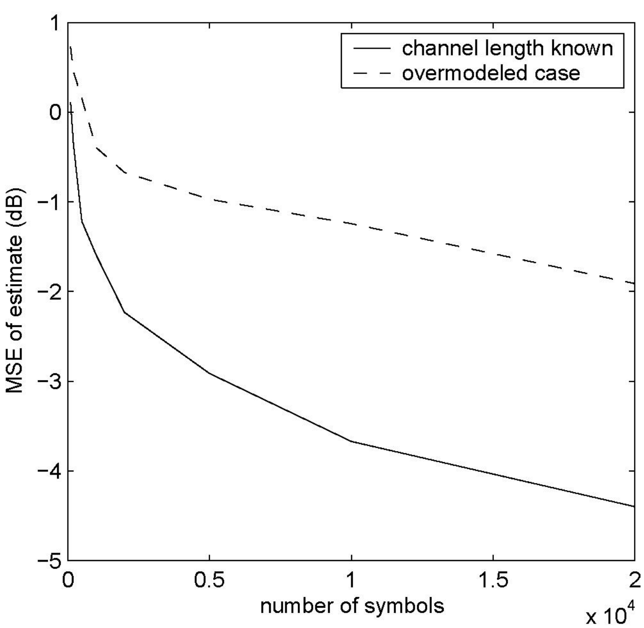

Fig. 2. Simulation Results

Additionally, we considered the performance of the algorithm when the receiver assumes that the channel is longer than it really is (i.e. when it does not have an accurate estimate of N_h). For this case, the MSE calculation (16) is done by appending a zero to the actual impulse response so that it has the same size as $\hat{\boldsymbol{h}}$. Running the same experiment as above, but using $\hat{N}_h = 11$, we arrive at the dotted curve shown in Fig. 2. As can be seen, a sizeable penalty is paid when the exact channel length is not known to the receiver.

5. CONCLUSION

We have considered blind channel estimation for orthogonal modulation. While equalization of orthogonal modulation has been previously studied, little attention has been paid to blind channel estimation. We extended a classical subspace-based blind technique [4] in a way that is suitable for use with orthogonal modulation. Unlike classical subspace-based blind channel estimators, however, our scheme does not require oversampling and/or multiple sensors. Future research could study the robustness of this scheme in harsh environments, for example in situations where the effective channel matrix is not full rank. In addition, future work could investigate alternative blind estimation/equalization techniques for this modulation, such as adaptive gradient descent-based algorithms.

6. REFERENCES

[1] A. G. Klein, D. R. Brown III, and C. R. Johnson, Jr., "MMSE decision feedback equalization of orthogonal multipulse modulated signals," in *Proc. Conf. on Information Sciences and Systems (CISS'04)*, Mar. 2004.

[2] M. K. Varanasi, "Equalization for multipulse modulation," in *Proc. IEEE Intl. Conf. on Personal Wireless Communications (ICPWC'97)*, Dec. 1997, pp. 48–51.

[3] J.R. Barry, "Sequence Detection and Equalization for Pulse-Position Modulation," in *Proc. IEEE International Conference on Communications (ICC'94)*, May 1994, pp. 1561–1565.

[4] E. Moulines, P. Duhamel, J. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," *IEEE Trans. Signal Processing*, vol. 43, pp. 516–525, Feb. 1995.

[5] Z. Xu and P. Liu, "A subspace approach to blind estimation of ultra-wideband channels," in *Proc. of 37th Asilomar Conf.*, Nov. 2003, pp. 1249–1253.

[6] L. Tong, G. Xu, and T. Kailath, "A new approach to blind identification and equalization of multipath channels," in *Proc. 25th Asilomar Conf.*, 1991, pp. 856–860.

[7] R. A. Horn and C. R. Johnson, *Matrix Analysis*, Cambridge University Press, Cambridge, UK, 1985.

[8] A. P. Liavas, P. A. Regalia, and J.-P. Delmas, "Blind channel approximation: Effective channel order determination," *IEEE Trans. Signal Processing*, vol. 47, pp. 3336–3344, Dec. 1999.